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THE VARIATION OF LATITUDE AT NEW YORK, AND A DETERMINATION OF THE CONSTANT OF ABERRATION FROM OBSERVATIONS AT THE OBSERVATORY OF COLUMBIA UNIVERSITY.

J. K. REES.

FOR POPULAR ASTRONOMY.

In 1892 the Columbia University Observatory completed its arrangements with Professor Em. Fergola, Director of the Royal Observatory at Capodimonte, Naples, for making a series of observations to study the variation of latitude. The Naples Observatory was selected because its latitude differed only 3 minutes of arc from that of the Observatory at Columbia. The regular Observatory of the University was then situated in the middle of the block bounded by 49th and 50th Streets and Madison and Fourth Avenues, New York City. This Observatory was on top of the library building of the University, and within 250 feet of the New York Central and New Haven Railroad tracks. In 1892, the trustees of the University purchased the new site on Morningside Heights, occupied at that time by the Bloomingdale Insane Asylum. The Director of the Observatory was able to interest the authorities of the Asylum, and although the property did not at once pass under the control of the trustees of Columbia, the Asylum trustees allowed the building of a little latitude Observatory on the grounds. The new site was further north than the old Observatory and also provided a place not disturbed by railroads. Here was erected in the fall of 1892 a small circular building of brick about twelve feet in diameter, having in its roof a proper slit for meridian observations. In the building was placed a zenith telescope built by Wanschaff, of Berlin, under the general direction of Dr. Albrecht, of Potsdam. Dr. Albrecht also consented to examine the instrument before it was shipped. The Wanschaff zenith telescope has an aperture of 80 millimeters, and a focal length of 1 meter. It is provided with two latitude levels. The illumination is electric. The instrument was in place and ready for observation about April 1st, 1893. Inasmuch as all of the officers connected with the Observatory were charged with daily work in the Uni-

No.	Name.	Mag.	Approximate Right Ascension 1875.			Declination. 1875.		
	Group I.		h	m	s	°	′	″
1	10 Camelopardi	4.0	4	52	18.38	60	15	22.871
2	*z* Tauri 102	5.0		55	37.51	21	24	33.418
3	λ Aurigæ 15	5.0	5	10	21.05	39	59	7.306
4	ρ Aurigæ 20	5.8		12	57.62	41	40	37.881
5	17 Camelopardi	6.0		18	22.13	62	57	33.609
6	119 Tauri	5.0		24	53.07	18	29	56.730
7	ξ Aurigæ 30	5.0		44	22.21	55	40	29.184
8	139 Tauri	5.3		50	14.27	25	56	9.812
9	2 Lyncis	4.6	6	8	35.58	59	3	10.456
10	μ Geminorum 13	3.0		15	23.89	22	34	31.892
11	ν Geminorum 18	4.6		21	32.44	20	17	20.780
12	8 Lyncis	6.0		26	15.72	61	35	15.588
13	ζ′ Geminorum 43	Var		56	41.66	20	45	5.658
14	17 Lyncis	7.0		58	25.41	60	59	8.508
	Group II.							
15	— Draconis (Pi. XII : 255)	6.	12	56	53.80	64	16	55.500
16	— Comæ Ber. (Pi. XII : 283)	6.7	13	3	39.15	17	30	56.709
17	20 Canum Venaticorum	4.6		11	56.14	41	13	52.193
18	23 Canum Venaticorum	5.6		14	42.77	40	48	25.799
19	83 Ursæ Majoris	5.4		35	59.70	55	18	53.429
20	3 Bootis	6.0		40	54.99	26	19	48.062
21	*v* Bootis 5	4.3		43	26.87	16	25	7.516
22	i Draconis 10	5.0		47	46.91	65	20	28.665
23	*a* Draconis 11	3.3	14	1	0.35	64	58	25.662
24	20 Bootis	5.0		13	50.29	16	52	49.744
25	— Bootis (Pi. XIV : 156)	6.0		34	17.27	54	33	51.853
26	34 Bootis	5.2		37	55.76	27	3	36.690
27	— Bootis (Pi. XIV : 178)	5.5		40	12.70	15	39	28.971
28	2H Ursæ Minoris	5.0		55	36.13	66	25	50.798
	Group III.							
29	ζ Draconis 22	3.0	17	8	25.72	65	52	7.124
30	— Ophiuchi (Pi. XVII : 94)	6.1		18	55.75	15	43	16.331
31	λ Herculis 76	4.8		25	41.17	26	12	22.406
32	*v*′ Draconis 24	4.7		29	42.95	55	16	13.085
33	*z* Herculis 88	6.0		46	47.08	48	25	43.411
34	— Herculis (Pi. XVII : 347)	6.1		56	1.67	33	13	9.913
35	b Herculis 99	5.0	18	2	16.83	30	32	42.835
36	— Draconis (Groom. 2549)	6.3		17	0.48	51	17	36.321
37	42 Draconis	5.1		25	37.39	65	29	10.324
38	— Herculis (Poulk. 2621)	6.4		31	32.59	16	5	33.331
39	— Draconis (Groom 2658)	6.0		39	50.60	62	37	31.854
40	— Herculis (Pi. XVIII : 203)	5.9		43	26.00	19	11	24.217
41	48 Draconis	6.0		54	38.16	57	38	58.554
42	— Vulpeculæ (Bradley 2409)	5.4	19	1	25.64	24	3	29.697
	Group IV.							
43	f′ Cygni 59	5.3	20	55	34.46	47	2	1.149
44	*v* Cygni 66	4.3	21	12	46.73	34	22	22.158
45	70 Cygni	5.1		22	15.56	36	34	27.286
46	ρ Cygni 73	4.1		29	16.80	45	2	23.302
47	16 Pegasi	5.3		47	22.54	25	20	15.817
48	13 Cephei	6.1		50	41.12	56	1	10.983
49	— Lacertæ (Pi. XXI : 405)	5.3	22	0	58.29	44	24	24.270
50	1 Lacertæ	4.6		10	31.28	37	7	36.349
51	25 Cephei	6.1		14	8.02	62	10	41.109
52	39 Pegasi	6.0		26	33.09	19	35	10.771
53	— Lacertæ (Groom. 3919)	6.1		48	22.90	39	42	39.015
54	*o* Andromedæ 1	3.6		56	10.39	41	39	16.161
55	2 Cassiopeiæ	5.8	23	4	23.78	58	39	18.234
56	*v* Pegasi 68	4.6		19	8.52	22	42	58.030
57	6 Lacertæ	5.	22	25	5.57	42	28	58.777
58	8 Lacertæ 2d	6.		30	18.71	38	59	16.172

versity, it made it impossible for one person to do all the work of observing. Moreover, during the summer the Astronomical Department conducts a summer class which occupies nearly the whole time of two of the three officers. It was decided, therefore, to divide the work of observing among three persons, and observations were made every clear night. During the first year and a half all observers lived about three miles away from the latitude Observatory, and it was necessary to make arrangements to sleep at the Asylum. Dr. Lyon, the superintendent of the Asylum, gave us every facility and comfort. Later on when the Asylum buildings were torn down to make way for the new University buildings, other arrangements had to be made.

In December, 1895, the latitude Observatory had to be moved to make way for one of the University buildings. The new resting place was several hundred feet north and west of the first position, to which it was connected by a carefully executed survey.

The two Observatories here and in Naples, being on very nearly the same parallel of latitude, were able to use the same stars. Four groups of stars were selected, seven pairs of stars in each group, and they were so arranged that the observations could be made for a determination of the constant of aberration by Küstner's method. The foregoing table gives the list of stars observed.

From May 1st, 1893, to July 1st, '94, the observations were made every clear night. After the latter date it was decided to observe four clear nights each half month, whenever possible. All of the observations were made by J. K. Rees, H. Jacoby, and H. S. Davis.

For the purpose of reduction our observations were divided into four series as follows:

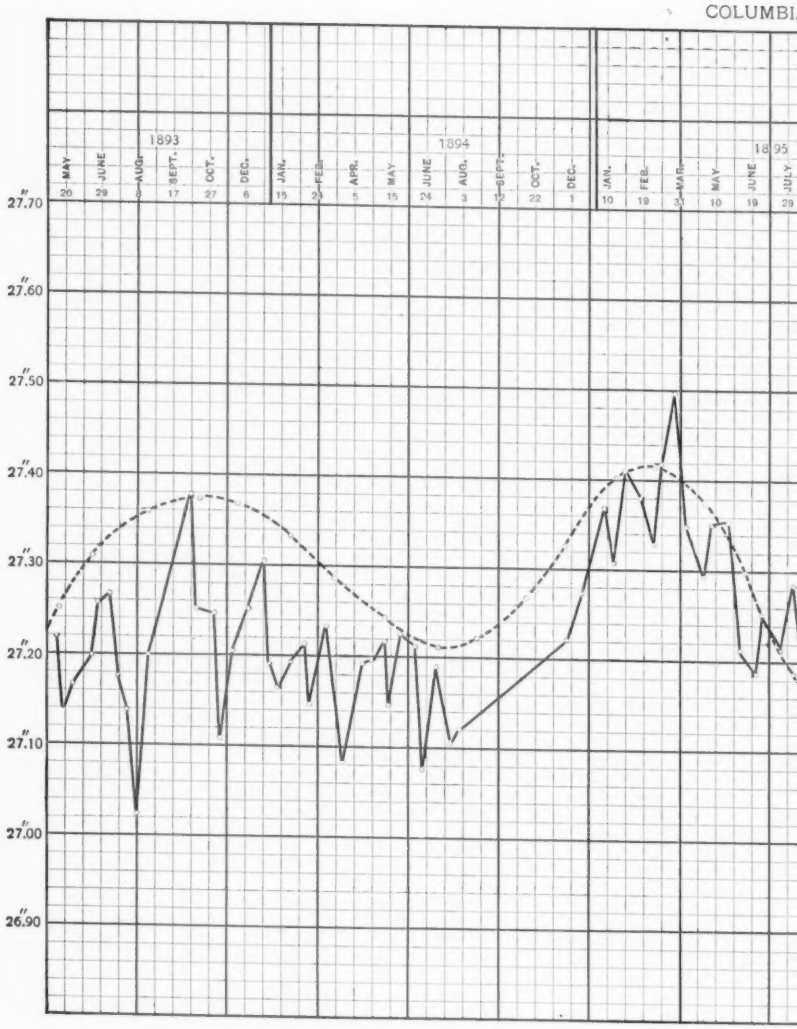
Series A.	April 24, 1892 to July, 1894	818 pairs	by Rees.
		302 "	" Jacoby.
		654 "	" Davis.
" B.	July 1894 to Jan. 1896	771 "	" Rees.
		310 "	" Davis.
" C.	Jan. 1896 to Jan. 1898	1065 "	" Rees.
		774 "	" Davis.
" D.	Jan. 1898 to Dec. 1899	951 "	" Rees.
		873 "	" Davis.
	Total pairs	6518	

This table shows the individual observers to have measured:

Rees	3605 pairs
Jacoby	302 "
Davis	2611 "
Total	6518

Date.	φ	$\varphi - \varphi_0$.	No. Pairs.	Date.	φ	$\varphi - \varphi_0$.	No. Pairs.	Date.	φ	$\varphi - \varphi_0$.	No. Pairs.
1893				1895				1897			
May 9	27.222	− .082	77	Aug. 10	27.295	− .009	19	Nov. 26	.089	− .215	37
19	.144	− .160	44	23	27.103	− .201	46	Dec. 10	.193	− .111	13
27	.162	− .142	42					26	27.100	− .204	22
June 8	.178	− .126	59	Sept. 15	26.943	− .361	26	1898			
17	.198	− .106	45	Oct. 1	.996	− .308	38	Jan. 10	.205	− .099	33
28	.259	− .045	32	18	27.111	− .193	55	26	.139	− .165	43
July 8	.262	− .042	30	Nov. 5	.119	− .185	50	Feb. 10	.177	− .127	53
19	.177	− .127	38	18	.063	− .241	36	Mar. 1	.246	− .058	88
26	.141	− .163	46	Dec. 2	.070	− .234	41	17	.169	− .135	25
Aug. 6	.029	− .275	46	1896				April 4	.285	− .019	33
Oct. 6	.377	+ .073	31	Jan. 14	.474	+ .170	37	19	.461	+ .157	22
15	.244	− .060	43	28	.476	+ .172	54	May 12	.365	+ .061	69
30	.241	− .063	44	Feb. 11	.542	+ .238	45	26	.429	+ .125	28
Nov. 9	.111	− .193	46	22	.600	+ .296	41	June 13	.501	+ .197	64
20	.204	− .100	48	Mar. 5	.560	+ .256	42	26	.539	+ .235	59
Dec. 3	.232	− .072	18	17	.627	+ .323	37	July 12	.488	+ .184	75
11	.255	− .049	28	28	.464	+ .160	21	26	.430	+ .126	23
23	.302	− .002	43	April 9	.477	+ .173	37	Aug. 12	.506	+ .202	32
1894				19	.367	+ .063	33	Sept. 6	.549	+ .245	47
Jan. 2	.192	− .112	36	May 5	.513	+ .209	56	17	.295	− .009	28
11	.168	− .136	65	18	.532	+ .228	27	Oct. 2	.488	+ .184	45
23	.198	− .106	61	June 4	.551	+ .247	47	15	.490	+ .186	42
Feb. 2	.214	− .090	56	20	.554	+ .250	30	Nov. 1	.485	+ .181	59
12	.148	− .156	59	July 1	.501	+ .197	48	15	.354	+ .050	40
24	.193	− .111	86	12	.437	+ .133	31	Dec. 1	.251	− .053	48
Mar. 5	.234	− .070	68	31	.458	+ .154	47	14	.264	− .040	28
21	.083	− .221	51	Aug. 19	.379	+ .075	57	1899			
April 1	.192	− .112	33	Sept. 9	.331	+ .027	23	Jan. 14	.138	− .166	36
11	.199	− .105	65	26	.240	− .064	27	23	.291	− .013	44
23	.218	− .086	83	Oct. 13	.135	− .169	35	Feb. 3	.151	− .153	28
May 2	.153	− .151	89	25	.142	− .162	53	23	.146	− .158	35
12	.225	− .079	62	Nov. 10	.193	− .111	59	Mar. 13	.060	− .244	24
29	.216	− .088	50	23	.118	− .186	23	26	.055	− .249	16
June 8	.079	− .225	68	Dec. 5	.232	− .072	37	April 6	.189	− .115	28
17	.193	− .111	64	15	.314	+ .010	15	26	.366	+ .062	48
July 1	.114	− .190	34	28	.219	− .085	27	May 9	.275	− .029	29
16	.120	− .184	38	1897				14	.194	− .110	39
Nov. 15	.226	− .078	39	Jan. 8	.345	+ .041	23	29	.267	− .037	61
29	.274	− .030	43	25	.214	− .090	41	June 16	.459	+ .155	11
Dec. 18	.367	+ .063	25	Feb. 5	.387	+ .083	28	30	.275	− .029	46
1895				18	.362	+ .058	53	July 14	.309	+ .005	48
Jan. 2	.305	+ .001	20	28	.378	+ .074	55	25	.391	+ .087	21
15	.412	+ .108	53	Mar. 10	.452	+ .148	38	Aug. 12	.418	+ .114	51
Feb. 2	.380	+ .076	39	23	.283	− .021	10	22	.458	+ .154	42
14	.332	+ .028	40	April 25	.583	+ .279	41	Sept. 4	.322	+ .018	63
26	.425	+ .121	35	May 21	.565	+ .261	73	30	.397	+ .093	31
Mar. 10	.500	+ .196	18	June 1	.376	+ .072	16	Oct. 17	.451	+ .147	35
21	.346	+ .042	24	17	.573	+ .269	48	Nov. 5	.361	+ .057	51
April 8	.298	− .006	30	July 2	.595	+ .291	44	24	.369	+ .065	53
20	.353	+ .049	48	20	.597	+ .293	26				
May 7	.354	+ .050	44	Aug. 4	.522	+ .218	70				
18	.213	− .091	47					Mean	27.304		
June 4	.184	− .120	28	Aug. 22	.639	+ .335	44				
13	.257	− .047	51	Sept. 30	.503	+ .199	97				
July 5	.217	− .087	45	Oct. 16	.324	+ .020	13				
16	.286	− .018	23	Nov. 4	.361	+ .057	38				
28	.222	− .082	46	16	.254	− .050	20				

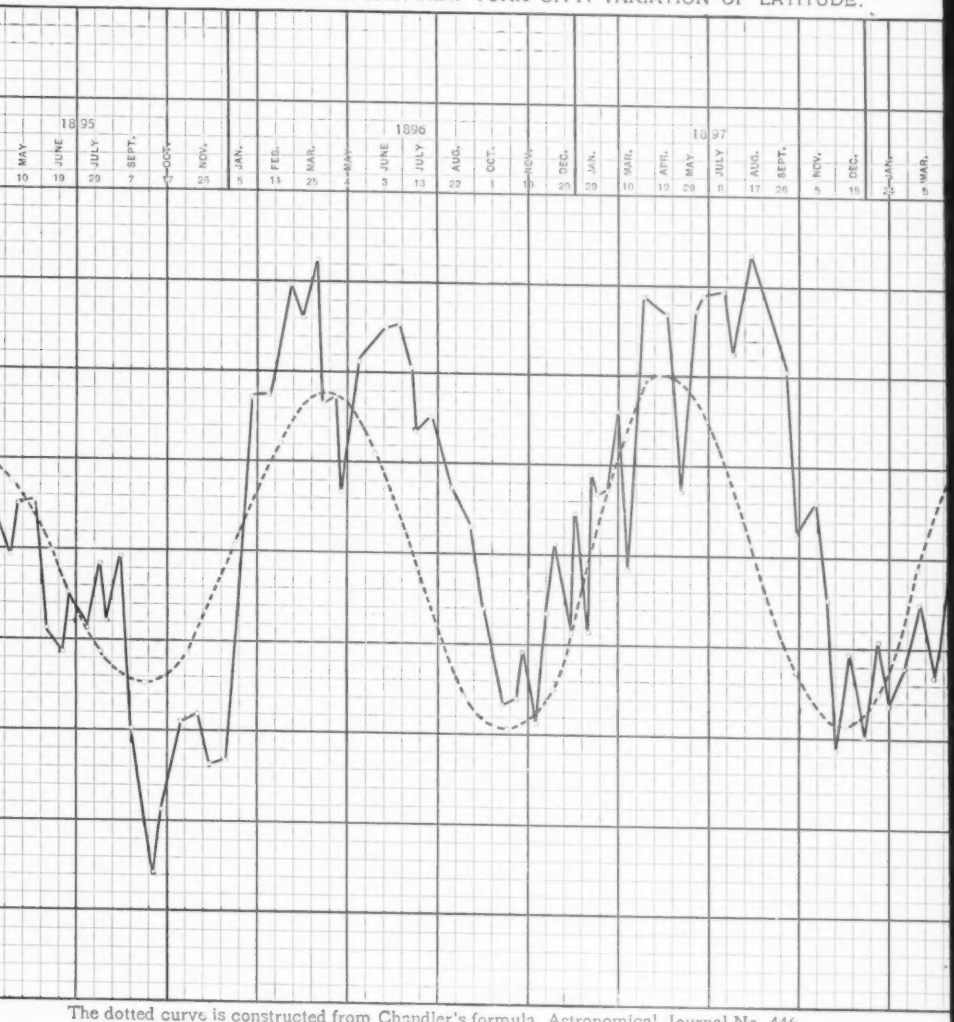
The record shows that observations were taken on 758 nights.


1893
1894
1895
MAY 20
JUNE 29
AUG. 8
SEPT. 17
OCT. 27
DEC. 6
JAN. 15
FEB. 23
APR. 5
MAY 15
JUNE 24
AUG. 3
SEPT. 12
OCT. 22
DEC. 1
JAN. 10
FEB. 19
MAR. 31
MAY 10
JUNE 19
JULY 29
27.70
27.60
27.50
27.40
27.30
27.20
27.10
27.00
26.90

The

PLATE VII.

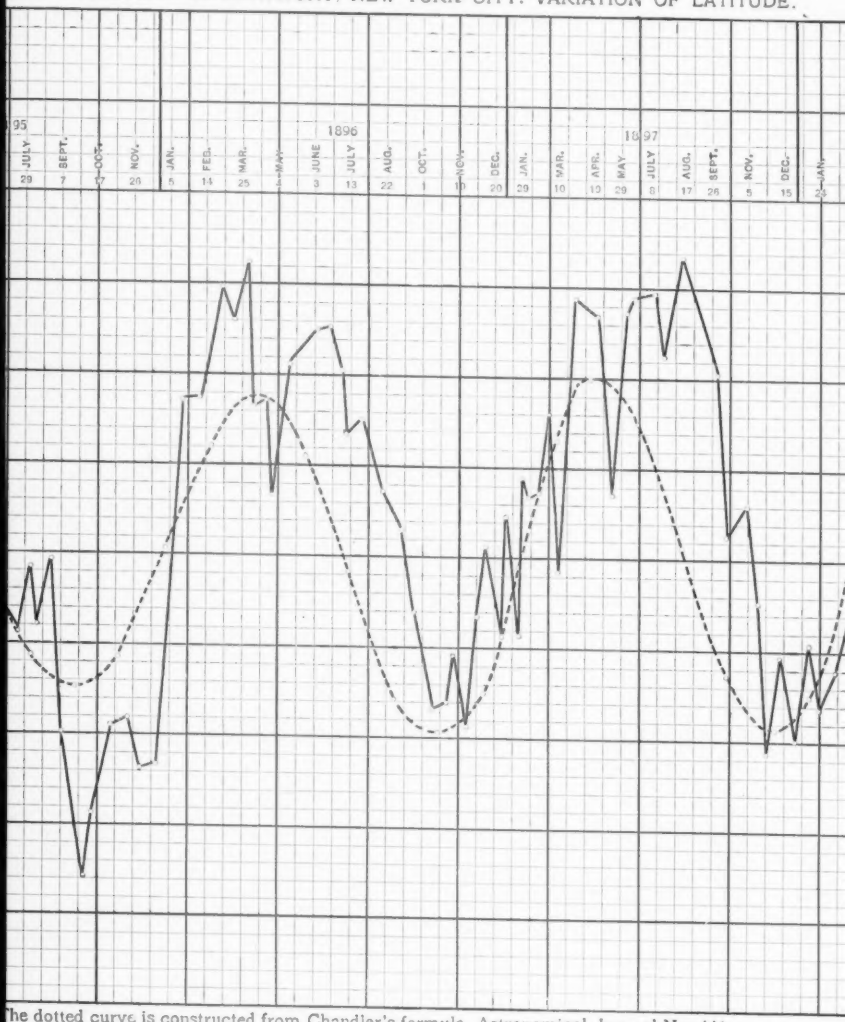
COLUMBIA UNIVERSITY OBSERVATORY, NEW YORK CITY. VARIATION OF LATITUDE.



The dotted curve is constructed from Chandler's formula, Astronomical Journal No. 446.

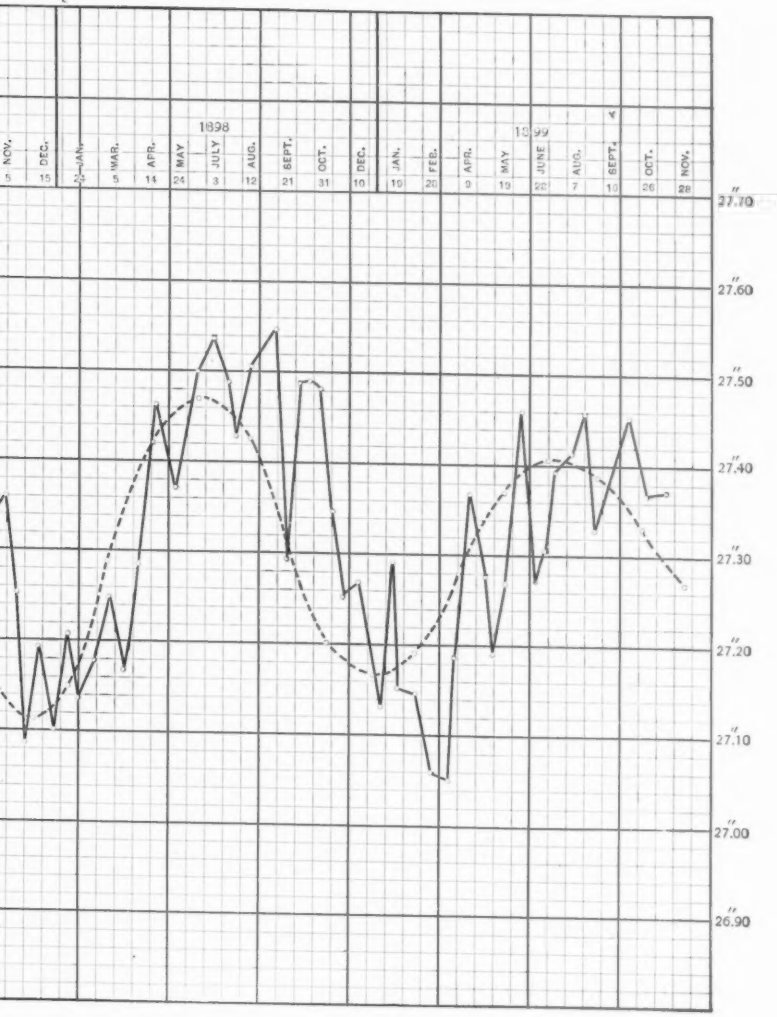
PLATE VII.

MBIA UNIVERSITY OBSERVATORY, NEW YORK CITY. VARIATION OF LATITUDE.



The dotted curve is constructed from Chandler's formula, Astronomical Journal No. 446.

TUDE.


1898
1899
NOV. 5
DEC. 15
JAN. 29
MAR. 5
APR. 14
MAY 24
JULY 3
AUG. 12
SEPT. 21
OCT. 31
DEC. 10
JAN. 19
FEB. 28
APR. 9
MAY 19
JUNE 28
AUG. 7
SEPT. 16
OCT. 26
NOV. 28
27.70
27.60
27.50
27.40
27.30
27.20
27.10
27.00
26.90

The computations give the following table* and curve which show the variation of latitude.

The curve required by Dr. S. C. Chandler's formula (*Ast. Jour.*, No. 446) is shown in the dotted line. From 1896 the observed epochs of maxima and minima follow the computed in time.

Dr. Chandler writes me that "on account of the different combinations of the observers participating at various times, there are doubtless constant differences of zeros in the observed curve at different epochs, which would bodily shift the latter up or down, over certain intervals. However the noticeable systematic differences of the observed and computed curves (notably from 1896 to 1898 where the observed epochs of maxima and minima distinctly follow the computed in time) are undoubtedly real, as well as extremely interesting. In my opinion they are referable to the fact that the annual ellipse is not stationary, but that its line of apsides is retrograding several degrees annually, as I have pointed out in my articles in various places (see especially *Ast. Journal* No. 446 where I have summed up the evidence). Of course the law of this revolution is not yet sufficiently demonstrated to justify its being incorporated in the formula which is hence based on a constant position of the ellipse ($\omega = 39°.6$); that is, the inclination of the major axis of the annual ellipse is assumed as 40° east of Greenwich, although the evidence is sure that it is backing around, so that it is now (1898-99) somewhat west of Greenwich. The fuller and more satisfactory investigation of this phenomenon must await the accumulation of two or three years' observations. By 1902 or 1903 I think it can be clearly apprehended, and formulated; not before. Another interesting fact is that your later observations confirm what your and other series earlier indicated, namely, that the dimension of the fourteen months' circular motion has progressively diminished since 1890. This is so well established that it was embodied in the formula some years ago." Nearly every element entering into this motion of the pole is variable.

The four series of observation gave the foregoing values of the Aberration Constant:

Series A.	20."4566	weight 18
B.	20. 4525	" 16
C.	20. 4695	" 26
D.	20. 4704	" 27

Taking the probable error of a single latitude observation as

* The previous publications will be found in *The Astronomical Journal* Nos. 401, 451, 474. In these papers the mean latitude is taken for each series and not, as in this case, for the whole time.

0″.16 gives Constant of Aberration 20″.464 ± 0″.006.

Fergola obtained a value, corresponding in time to our Series A, of 20″.53, using the same stars and the same methods of reduction.

Harkness in his work on "The Solar Parallax and its related Constants" (1891) gives the values of the constant of aberration determined between the years 1817-1888, and from his discussion of these values gets 20″.466 ± 0.011. Newcomb in 1895 published his book on "Astronomical Constants" in which he gave the values of the aberration constant down to 1894. He divided the results into two classes: A, standard Pulkova determinations, and B, other determinations. The separate results he weighted and obtained

Mean result from A	20.493 ± 0″.011
" " " B	20.463 ± 0 .013

The first result is almost "identical with that found by Nyrén" in 1883: this was 20″.492 ± 0″.006 and was not included in list A or B.

These results were all obtained on the assumption of a constant latitude. Dr. Chandler has rediscussed and obtained quite different results. The Paris Conference of May 1896 adopted the value of 20″.47.

Doolittle's results gave him a mean value of 20″.580 from observations made 1896-1898: Fergola obtained, as stated above, at Naples the value 20″.533. Finlay at the Cape of Good Hope obtained the value 20″.57 and at Berlin and Strassburg have been obtained the values 20″.511 and 20″.475.

The value 20″.47 corresponds to the value of solar parallax of 8″.8033, while 20″.53 gives 8″.7773.

The differences in the various results obtained indicate plainly that we must wait some time longer for a definitive value of the aberration constant.

PHOTOGRAPHS OF THE ZODIACAL LIGHT.

A. E. DOUGLASS.

For Popular Astronomy.

The experiments which resulted in the accompanying photographs of the zodiacal light were the outcome of long continued interest in the subject of the gegenschein and zodiacal light and a desire to render less fatigueing and more accurate the observation of those faint lights. Contours of the zodiacal light drawn

by hand on about two hundred different nights had shown the defects and difficulties of that method of recording observations. Perhaps the chief of the defects was the almost invariable interference of stellar light; yet that was not the only important one, for the very first result of these photographs was to show that in estimating a visual contour one is apt to mistake ease in distinguishing the zodiacal cone at any given point for actual intensity at that point and therefore the contours of equal brightness are prolonged along the axis too far away from the Sun and horizon light. After seeing the photographs this erroneous tendency was actually found to exist.

The inherent difficulties in the old method lie in the long time required to get perfectly acquainted with standard reference stars and the advantage, that amounts almost to necessity, in making records in the dark. Even an old hand at this work will frequently be obliged to make use of some star whose name he does not know and will have to describe its place, as well as that of the zodiacal light, on paper which he can barely see resting in his hand.

The first effort to improve upon the method of observation consisted in designing a machine which could automatically record the contours directly on a star map, and a rough model of this contrivance gives entire promise of success. But as this was not put into actual practice attempts were made to succeed by photography. Many attempts have been in this line elsewhere, but without success. Of these one was by myself while in South America (in 1891, I think) and another has since then been made at that same station. The early work at Flagstaff was equally without result. But after repeated trials success was at once attained when the very simple idea of using ordinary positive visual eyepieces, which combine very short focus with relatively large aperture, occurred to me and was tested and from that time on constantly improving results have been obtained. The most successful lens and the one by which the illustrations for this article were taken, is a combination lens, put together and mounted by Mr. Cogshall, who has done practically all of the photographic work. This lens seems to give a combination of flat and large field and very great light-transmission that surpasses any other apparatus that we have tried, and among those tried the one marked "Clark; Special" in the list below is a "solid" achromatic lens cut on the Fraunhofer curves. The following list describes briefly the various lenses tried by Mr. Cogshall and gives a summary of our experiments:

Make.	Focus.	Aperture.
St. Louis house; name unknown	16 inches	2 inches.
Same, with telescope lens to shorten focus	10 "	2 "
Rochester Optical Co. Rap. Rect	8 + "	¾ "
H. & E. J. Dale, London. (A hand camera)	5 + "	¾ "
" W. A. C. Pinhole" Camera	3 "	¼ "
Zeiss Planar Series Ia. By Bausch and Lomb	1⅜ "	5/16 "
Eyepiece; Brashear	1 "	½ "
" "	1 "	¼ "
" "	1 "	⅛ "
" "	½ "	¼ "
" "	⅜ "	⅛ "
" Clark	2 "	¼ "
" "	2 "	⅜ "
" "	2 "	½ "
" " "Special"	2 + "	⅜ "
"Combination;" part made in Paris and part by Clark; combined by W. A. C	1.3 "	½ " or 0.4 (⅜) for best work.

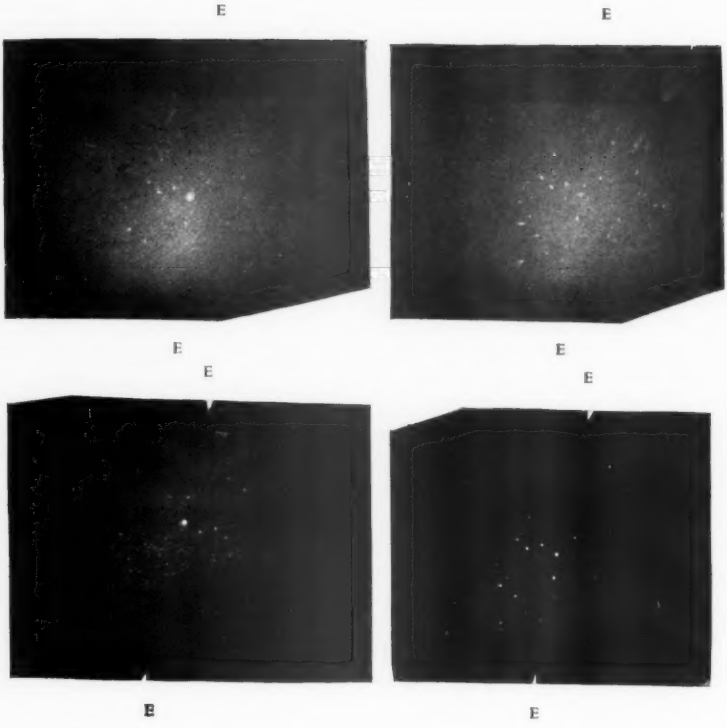
After obtaining real photographs the first step was to make sure that there was no deception. Some of the views taken as long ago as May, 1899, were really exceedingly good but had some trifling defect, which threw a slight doubt on their genuineness. But they were repeated many times. Finally a very thorough test for ghosts or other concentration of light in the fields of the chief lenses used, was made by trial exposures on such objects as landscapes and ruled paper, by daylight, the side of a house by moonlight, and the sky, by day and by night. No genuine irregularity or concentration of light could be found in the fields over an area considerably greater than the entire portion of sky shown in our illustrations, and the photographs were therefore accepted as real. (Let me here explain that while taking the photograph of October 7, a pasteboard tube supporting the lens, projected too far inside the camera and cut down the field; this effect shows conspicuously in the original negative.)

The first conclusions drawn from these photographs are that the axis of greatest density is very indefinite and that the photographic contours of the apex of the zodiacal cone are far more rounded in form than the visual ones are usually represented; and, as stated above, I am inclined to think this same roundness is really true of the visual outlines.

Another conclusion and one of much interest to the experimenter, is one that cannot be derived from our illustrations but has been found to hold true on other photographs. It is that with this form of camera the zodiacal light makes an impression on the sensitive plate more readily than equally bright regions of the Milky Way.

The horizon light does not appear to effect these photographs.

PLATE VIII.



October $7^{d}\ 16^{h}\ 20^{m} - 17^{h}\ 15^{m}$

October $14^{d}\ 16^{h}\ 15^{m} - 17^{h}\ 20^{m}$
and October $15^{d}\ 16^{h}\ 16^{m} - 17^{h}\ 20^{m}$

Photographs of the Zodiacal Light in Leo.

The letter E at the top and bottom of each photograph indicates the direction of the Ecliptic.

Of course they are not allowed to continue when the horizon light is very strong. Nor am I sure that the atmospheric absorption materially affects the intensity of the light near the horizon, since lessened exposure on the lower edges will account for the faintness in that region.

Finally in these photographs of the zodiacal light in Leo, at a distance of about 8° from the ecliptic, the intensity of the light fades much more rapidly on the southern side than on the northern. This effect may be partly due to atmospheric absorption but as the axis of the cone in this instance is inclined over 70° to the horizon, I am inclined to believe this a real effect due to the form and position of the great lenticular mass of particles which cause the zodiacal light.

LOWELL OBSERVATORY,
February 26, 1900.

ATTRACTION AND THE FIGURE OF THE EARTH.

W. W. PAYNE.

From time to time so many questions are asked about the law of gravitation, its history, its development and its applications, that it has seemed best to try to answer some of them, in a general way, and in popular language, for the benefit of some readers who have not the knowedge of the higher mathematics that will enable them to read the works of standard authors on these difficult themes.

We have decided to do this now because we have seen lately, more then ever before, something of the waste of time and money, by people of good minds, who apparently have not pursued the branches of mathematics and physics far enough to fit them for the study of such questions, either for their own benefit or for the instruction of others; much less are they qualified to write books on new theories of attraction, or to set up claims that the law of gravitation is not true, because some of its applications are not proved, or seem to be false, as determined by methods of algebra, geometry and trigonometry.

The history of the law of gravitation is so closely connected with that of the figure of the earth that the consideration of the one necessarily involves that of the other; so we begin with a brief account of the views held by some ancient people of renown about the shape or figure of the earth.

In very early times the prevailing view seems to have been that the surface of the earth was flat; that the earth itself was bounded by an ocean of unknown extent, and that the great sea was limited by the visible horizon; that the sun rose in the morning out of this great ocean and each evening set in it. It is but natural to think that the first people of earth should believe that the relation of things on so large a scale as that visible to the eye should indeed be exactly as they appear, and that the earth and the heavens were the whole of the universe.

The Chaldean opinion of the shape of the earth was, however, unexpectedly peculiar. They believed that it was like the half of an orange, with the curved side upwards, but it does not appear on what the flat side of the orange rested. The Chaldean astronomers supported their belief, as they supposed, by scientific argument, as we are told in Lenormant's Chaldean Magic, page 150. Such proofs as they could frame would not be certain, however, because the scholars of those times did not have sufficient knowledge of physical laws for a sound basis in reasoning on such questions. Some writers are inclined to make an exception to this view in the case of the Chinese whose definite records of eclipses go back as far as 800 B. C., because they knew, it is claimed, that the sun and the heavenly bodies were visible to people towards the east sooner than to those further west. This they must have known could only be due to the curvature of the earth. This view is somewhat strengthened from naked-eye observations of the sky, day or night. These early people must have noticed its spherical shape by day, and they would also observe the daily motion of prominent star groups, we call constellations, around a particular point in the heavens, which we now call the celestial pole; but whether or not they would infer the globular form of the earth from these motions, and from the change of altitude of the stars above the horizon to one traveling north or south, has been a matter of some doubt. It seems to us that thinkers and acute observers, at almost any time in human history could have inferred so much, when we remember some of the astonishing things they did learn under very unfavorable circumstances. And yet we are in doubt about this, if we go back in history no further than about 600 years, in the time of boasted learning among the ancient Greeks. Thales of Miletus, 640 B. C., Anaximander, 570 B. C., and Anaxagoras, 460 B. C., all claimed that the earth was cylinderical in shape, its height being three times its diameter. They found place for all the land and water of the earth on the upper base of the cylinder. Plate, 400 B. C.,

said the earth is a cube; but Aristotle of Thrace, 384 B. C., was probably among the first of the early scholars to advance the theory that the earth is a sphere..

The reader of history knows that Egypt was in the lead, in these early days, in most scholarly pursuits. Before the founding of the Ionian school, which was the beginning of the study of mathematics under the Greek influence, Thales was pursuing the studies of geometry and astronomy in Egypt, so it becomes of interest to learn what we may about the opinions held by the Egyptians regarding the form of the earth.

About thirty-five years ago a so-called hieratic papyrus which forms part of the Rhind collection in the British Museum was satisfactorily read. This writing throws some light on the advancement of the Egyptians in very early times in arithmetic and geometry. It is believed by those who have read it, that this manuscript was written by an Egyptian priest, named Ahines, somewhere between 1700 B. C. and 1100 B. C., itself being only a copy, with corrections, of an older writing whose date was about 3400 B. C. The title of this old manuscript is "Directions for Knowing All Dark Things." For a brief review of the arithmetic and geometry there presented, see Ball's Short Account of the History of Mathematics, p. 3, and following. From Ball's review, it is evident that the knowledge of these two branches of mathematics in Egypt was sufficient 2000 years B. C. to have led scholars of that time to make useful applications of them to the study of the figure of earth, if their minds had been so directed. But it is one thing to have knowledge of a principle in mathematics, as proved by a course of reasoning, as a proposition is demonstrated in plane geometry, but it is quite another and a different thing to be able to apply a principle so learned to its concrete uses, though it be simple in kind, and sometimes quite obvious in relation. As we read of the early history of the Egyptians, this seems to be strikingly true in what they know of elementary science—so much so that it is hard to account for the utter lack of ability to apply their reasoning powers to the study of the shape of the Earth, when they had done so well in the elements of pure science and philosophy.

It may be possible to explain part of this enigma by the fact that the scholars of those very early times were the priests whose influence with the people was very large, and whose privileges secured to them by the ruling classes were exceptionally great. These priests were much occupied with idolatrous worship, which filled Egypt with it massive temples, its numberless objects of

worship, and its endless ceremonies. These things being so, they would hinder rather than promote the advance of truth or science in any important way. In proof of this we have only to remember that there were gods for the Sun, dawn, sky, Earth, water, storm, and indeed as many other things on the Earth, under it, and in the heavens, as one may be pleased to enumerate. The god, Varuna, who was king of the sky, is especially interesting to us just now, because we read that,

"Varuna stemmed asunder the wide firmanent; he lifted up on high the bright and glorious heaven; he stretched out apart the starry sky and the Earth." * * * * "This Earth, too, belongs to Veruna, the king of this wide sky with its ends far apart. The two seas (the sky and the ocean) are Veruna's loins."*

These are a few sentences from the sacred songs of the Vedas, which were chanted often by Egyptian priests in the ears of the people 1500 years B. C. They were written in Sanskrit, until recently almost an unknown language, which we now learn bears a near relation to our own English tongue. The people who joined in such worship were the ancestors of Aryan races, the great family to which the Anglo-Saxon belongs. We are interested in the early thoughts of their intellectual life and growth, especially because we learn from the above quotations, and others like them, that they believed the Earth was something like a cylinder in form, for its "ends were far apart." This seems to be the source of the belief later held by the Greek civilization, for it was the same thing in the time of Anaximander, and modified only a little by Plate, to that of a cube, because, philosophically, the cube was a more perfect figure.

When we come to the later Indian pictures, ideas seem to be a little more concrete and less poetical, and strangely enough, when we learn that the Earth was then thought of as a large shell supported by "Elephants (representing strength), the elephants being supported on a tortoise, (which represented infinite slowness)." We are not told on what the tortoise rested; it doubtless needed none, for it moved infinitely slow.

This must suffice for the early views of the form of the Earth. We next consider the spherical hypothesis.

*See The Dawn of Astronomy, Ch. I., by Norman Lockyer.

ORIGIN OF THE LUNAR FORMATIONS.*

The characteristic difference between the *maria* and the larger craters, like Clavius, Ptolemy, and Schickard, is that every crater is surrounded by a high mountain ridge. In the case of the older and larger craters, when not too far destroyed by subsequent melting, the interior of the walls show a terraced structure due to the past tidal action. The *maria*, on the other hand, are usually surrounded by low shores. When they are high, as in the case of Crisium and Humorum, the shore is a plateau rather than a mountain range, although the northeastern boundary of Humorum is somewhat mountainous for a short distance. The northern boundary of Mare Imbrium is distinctly a plateau, and the only apparent exception to the general rule seems to be in the case of the Apennines and Caucasus. But even here it appears that these ranges originally formed the edge of a plateau that has been somewhat tipped out of its former horizontal position. According to this view, therefore, the Apennines and Caucasus are merely the curved edge left to the original solid surface by the molten flood, which, welling up from the interior, melted and destroyed everything before it, as long as the supply of heat caused by the local subsidence lasted. Portions of the original surface, only partially destroyed, are seen in the regions about Archimedes and Autolycus.

As soon as the surfaces of the *maria* had solidified, fresh compression of their molten interiors began, and a second era of crater formation was inaugurated. These craters necessarily differed from those of the earlier period in one respect, however, and that was that while the walls of the earlier craters were light colored, those of the secondary period were of the same color as the material of the surrounding *mare*. Thus, of two craters located side by side in the middle of a *mare*, one may be light and the other dark. In such a case the former one survived the flood, the latter was formed subsequently to it. This secondary period seems to have extended to the present time, for, as we have already seen, small craters are even now being constantly produced, as in the case of the interior of Plato. Some craters, like Aristarchus, and certain small craterlets, are intensely brilliant; but this is an extraneous circumstance, as we shall presently see, having no bearing on the age of the crater.

The hypothesis is frequently maintained that the *maria* were originally covered with water. There seems to me no sufficient

* Continued from page 153.

evidence to be found in support of such a conclusion. As shown in my paper published in *Astronomy and Astro-Physics* for 1892, Vol. XI, p. 778, it is probable that the lunar atmosphere was never very dense, perhaps not exceeding one inch in pressure. Under these circumstances the evaporation from any large body of water exposed to the Sun's rays would be very rapid, and the condensation at night equally so. This more than tropical precipitation could hardly have failed to leave very conspicuous evidence of its former existence in denuded slopes and deep ravines. Moreover, the *maria*, although frequently in communication, are placed at very different levels, as has been often pointed out. As the water gradually dried up, or was absorbed, and the upper seas emptied into the lower ones, channels would have been cut connecting them. Nothing of the sort is seen, however, although these would be the most favorable regions in which to observe them. In short, the evidence of the action of water upon the Moon is certainly very much less marked than upon the Earth, and seems quite inadequate to support the hypothesis that the *maria* were ever flooded.

The lunar craters may be divided according to their appearance into six classes, distributed into two periods. *Craters of the first period.* These all have bright walls. (a) Unaltered craters. These have high sharp walls with rough interiors. Copernicus, Tycho, Arzachel. (b) Partially submerged craters. These have sharp walls but smooth interiors, which may be either dark or light. Plato, Archimedes, Ptolemy, Schickard. (c) Softened craters. The whole crater floor and walls seem to have been softened and flattened. Posidonius, Cassini, Gassendi. (d) Submerged craters. Only portions of the rims of these craters show above the surface of the *maria*. Large crater near Flamsteed. Numerous craters in nearly all the *maria*. *Craters of the second period.* All craters having dark walls belong to this class. (e) Dark craters shaped like Copernicus, but on a much smaller scale, rising in several of the *maria*. Lambert and other craters in Mare Imbrium. (f) Craters with smooth floors and walls, the latter having low exteriors and deep interiors. Most of the smaller craterlets found in the *maria* belong to this type. Helicon, Bessel, and Horrocks are among the larger examples.

Owing to the sinking of the convex surface of the *mare* a compression will arise, and ridges be formed. These are usually concentric, as in Maria Imbrium, Humorum, and Procellarum, but sometimes they pass through or near the centre, as in Mare Serenitatis. When these ridges have sharp crests they become less

able to resist the internal compression, and craterlets often force their way through in such places. This compression, as we have seen, is the probable cause of the central ridge or peak found in many of the unaltered lunar craters, the weight of the surrounding mountain wall being an efficient aid in producing the requisite compression. As in the case of the ridges in the *maria*, a craterlet is sometimes found in the summit of the central peak. Under these circumstances the appearance is occasionally quite similar to that of the smooth, high, truncated cone of a terrestrial volcano, as in the case of Timocharis; nevertheless, I do not believe that volcanoes proper are to be found upon the Moon. Certainly no evident instances of such formations have ever been clearly seen. Nor is it evident how the necessary explosive force could be obtained, unless we assume the former presence of large bodies of water upon the Moon.

Besides the generally recognized *maria*, there are several similar smaller formations which have not as yet received any special designation. The most important one is of about the same shape and size as the Mare Crisium, but of rather lighter color. It is situated nearly due west of it, and is only brought into view under a favorable libration. Another is situated south of Mare Humorum and west of Schickard. Had the melting of the original surface been a little more complete, it would undoubtedly have been designated as a *mare*. On the other hand the large nameless plain southeast of Schiller is clearly a ruined crater, perhaps the largest to be found upon the Moon. Another small unnamed *mare* is situated southwest of Humorum, between it and Tycho. The very elongated crater of Schiller is really composed of a series of craters, the dividing walls between which have been broken down by internal heat.

Among the most important of the smaller formations upon the Moon are the rills. The rills proper, as distinguished from the river beds already described, are undoubtedly cracks in the lunar surface, formed at a time when it was too small for the interior, but when the fluid contents was so far removed from the surface that but little of it was able to escape and relieve the pressure. The rills are therefore of comparatively recent formation. They are rarely if ever found among the primary formations, unless these have been melted or softened by the subsequent application of heat. They are never found near the poles where the tidal action is small, although seen near other parts of the limb. Only three minute rills, therefore, are found in the Mare Frigoris, although they abound in the other *maria*. The

most northern rill known is in latitude + 57°, north of Aristoteles, the most southern one is in latitude — 35° south of Ramsden. They are particularly numerous in the southeastern quadrant of the Moon, and comparatively rare in the southwestern. They are frequently concentric with some *mare* or partially submerged crater. Striking illustrations of this are found just to the west of Mare Humorum where three concentric systems of rills occur. The appearance of this *mare* is as if a thin skin had formed over the liquid surface, and had then been broken and crumpled, and drifted to the eastern side. Towards the centre and on the western side of Humorum are some long ridges, the western one concentric with the *mare* and the rills. On the eastern side are some deep rills also concentric with the *mare*. The paraffine crater, Figure 15, shows a minute ridge stretching from the nearer side towards the centre. Later a rill formed upon the further side concentric with the crater, thus showing that the surface was subjected first to compression as the floor of the crater fell, and later to tension as it contracted and receded from the crater walls.

The direction of the rill in many cases bears no apparent relation to the surrounding formations, and seems to be the result of a general contraction of the lunar crust. When it traverses a crater it is often evident that the latter is the earlier formation, as in the case of Hippalus. On the other hand, in the case of Ramsden the crater seems to have been formed later. Some extremely broad and deep concentric rills are found in the interior of Wurzelbauer. The appearance of a crumpled skin seen in the Mare Humorum is also found in other formations, as in J. Herschel.

An extremely rough and broken surface is found just to the north of Sinus Iridum. Few craters are to be seen, and it looks as if it might have been a part of the original surface of the Moon, before it had been pitted with craters or fused into *maria*. But what makes this region particularly interesting is that we have here a clear view of a section of the surface some two miles in depth, cut by the melted lava that formed the Sinus Iridium. An examination of this section shows a nearly vertical wall, in places perhaps overhanging, apparently composed chiefly of objects like huge boulders, measuring several thousand feet in diameter, but separated from one another here and there by interstices forming caves of surprising dimensions. As seen under favorable conditions, at Arequipa, the appearance was not unlike that of a piece of wood broken squarely across the grain.

The structure of the wall is entirely different from that of the Apennines, which bound the Mare Imbrium on the west. Neither does it resemble the smooth sloping terraced interiors of the larger craters.

Among the more noteworthy of the minor lunar features may be mentioned the following formations. To the north and also to the south of Copernicus are found a number of spindle-shaped cuts in the surface, like very elongated craters. They are on the average about two miles long by half a mile broad, and very much resemble in shape the mark that a bullet might make in penetrating obliquely a somewhat vicous surface. Scattered among them are a number of smooth rounded mounds, some circular and some elongated. Some of the latter look as if they would exactly fit into the spindle-shaped cuts. Similar cuts are found in Cassini. To the north and west of Copernicus are a series of irregular elongated markings formed in part of craterlets, and due perhaps to the escape of gases through the formerly viscous surface from a submerged crack or rill. Similar markings are found to the southeast of Schiller. A little over one diameter due east of Gassendi is a curious ring of isolated elevations surrounding a central peak. A little north of Cavalerius is situated a very striking black peak.

A series of well marked parallel grooves each groove several miles in width, forms a characteristic feature of certain portions of the Moon's surface. They are well seen in the vicinity of Pallas, and lie in a northeast and southwest direction. They extend from somewhat to the north of Hyginus as far south as Albatagnius. A less marked series of parallel grooves lies southeast of Sinus Iridium, and another near the south pole. A large ruined nameless crater of oval shape, and exhibiting a curious spiral arrangement of supporting ridges is to be found lying between Wurzelbauer and Heinsius. Although lava streams are not usually clearly defined upon the Moon, one may be seen where Wargentin overflowed its crater on the northeast, flooding the surrounding region. This overflow probably prevented the lava from rising higher in the adjoining crater north of Phocylades, whose interior is on the same level as the surface of Wargentin, and several hundred feet higher than the level in Phocylades proper.

The last feature that we shall refer to upon the Moon is the system of bright streaks conspicuously surrounding six of the larger crater formations, and also visible around many of the smaller craters. In the *Astronomische Nachrichten*, Vol. CXXX,

p. 225, I pointed out certain facts regarding these streaks, derived from my observations made at Arequipa, of which the following are the most important. Hundreds of these streaks exist upon the Moon, but in the great majority of instances they are found to issue from minute intensely white craterlets. These craterlets are seldom over 1″ in diameter, and are usually much less. The streaks are very brilliant where they issue from the craterlet, but broaden and grow fainter as they recede from it. Their maximum breadth seldom exceeds 5″, and their length usually varies from 10″ to 50″. Those streaks which do not issue from minute craterlets usually lie upon or across ridges, or in other similarly exposed situations. Frequently a number of the craterlets producing streaks are found to be arranged in the same direction in which the streaks lie. In such a case, before one streak comes to an end another will begin, thus forming a nearly continuous white band. A few of these white bands extend for over one thousand kilometers. The nature of the individual streaks is seen very clearly in the crater Pitatus. It is also well shown in Copernicus, but does not become clearly visible there until the thirteenth day after new Moon. The combination of the streaks to form a white band is most readily studied in the conspicuous marking stretching from Tycho to Mare Nectaris. Under favorable atmospheric conditions it can be seen in the two parallel bands extending from Tycho towards Kepler, and in the band stretching from Menelaus across the Mare Serenitatis.

The visibility of the streaks can be studied very satisfactorily from photographs. They become visible in general about twelve hours after sunrise and brighten for one or two days. They remain visible until about the same period before sunset. Latitude has little if any effect on the time of their appearance. They are probably due to the same cause that produces the variable white markings and spots in Plato and about Linné and Schröter's Valley.

Since the pressure of water vapor at the freezing point is 4.6 mm., it is manifest that, unless the pressure of the lunar atmosphere exceeds this figure, water cannot exist upon the surface of the Moon at all in a liquid state, no matter what the temperature may be. Above the freezing point its condition will be wholly gaseous, below the freezing point it may be wholly gaseous, or it may be partly solid and partly gaseous. Since it is doubtful if the pressure of the lunar atmosphere exceeds 0.5 mm., it is clear that water as such cannot at present exist

upon the surface of the Moon. There is no reason, however, why the white markings above mentioned may not be due to ice crystals, either suspended in the air or deposited upon the ground. That the white substance cannot be frozen carbonic acid is certain, on account of the enormous pressure that would be involved, even at extremely low temperatures, to keep it in the solid form. Whether the increase and diminution in size of the white markings, and their transportation from place to place is due to the bodily transference of the crystals by means of the lunar winds and the subsequent melting of the crystals, or whether the water is transported in the gaseous form and later deposited as hoar frost, is not as yet determined, but I am inclined to believe, from the appearances noted about Schöter's Valley and about Messier, that actual clouds due to ice crystals really exist upon the Moon. That so rare an atmosphere should be able to transport the ice crystals at all seems at first sight very remarkable, but it is possible that the crystals are very minute, as would indeed probably be the case when condensed in such an atmosphere. In considering the plausibility of the explanation that the white substance causing the bright streaks and craters is really hoar frost, an idea which was, I believe, first suggested by Mr. Ranyard, (Knowledge, 1890, Vol. XIII. p. 130,) we may call to mind the observations already described in the section devoted to Linné. It was there pointed out, not only that the white area was diminishing in size from year to year, but also that there was a large periodic variation from day to day, the micrometric measurements showing that as lunar midday approached the spot grew smaller, reaching a minimum soon afterwards, and then again increased in size with the approach of night, much as the polar caps of the Earth increase and diminish with the seasons.

In order to obtain a satisfactory explanation of the bright streaks, we may start with the observed fact that those craterlets producing streaks which are situated in the vicinity of a prominent streak centre, as Tycho, for instance, are distributed not quite uniformly, but with a tendency to take the form of radial bands. It is very evident that when those craterlets near the center become active, they will give rise to a wind blowing away from Tycho in all directions. As this wind proceeds outwards, it will be reinforced by the wind from the various active craterlets that it encounters upon the way; there will therefore, in general, be no opportunity for diverse currents, and the bright streaks must necessarily lie radially as we observe them. As we

have seen in the previous chapters, the activity of the craterlets sometimes, as in the case of Schröter's Valley, begins shortly after sunrise and ends before sunset. In the case of Linné, on the other hand, the activity apparently lasts for a short time into the night.

If the supply of ice crystals is limited, as must certainly be the case upon the Moon, and the country over which they are blown is rough, they will settle almost exclusively in the crevices and hollows between the rocks. Moreover, this tendency will be further increased, as we see upon the Earth, since the Sun will tend to evaporate those in the more exposed situations. Since the shadows are perfectly black upon the Moon, the crystals upon which the Sun does not shine will reflect no light. Therefore at sunset and sunrise the bright streaks should be comparatively inconspicuous, showing best under a high illumination. The phenomenon should be quite independent of the position of the Earth, upon which depends the phase of the Moon. This we find to be the case, since the streaks first appear clearly a few days after sunrise, regardless of other circumstances as to whether the Moon is a crescent, gibbous, or full. Moreover, if we view the dark side of the Moon three or four days after the Moon is new, the bright streaks will be clearly seen by earthlight, although invisible both before and after, at sunset and sunrise. When the crystals are exposed to the Sun they will appear bright even at sunrise, as in the case of Aristarchus. In the crater of Messier, on the other hand, they do not begin to form until the Sun is well up, so that the interior does not become bright until three or four days after sunrise.

The streaks are sometimes extremely narrow. Thus, in 1893, six were seen radiating from invisible craterlets situated upon the lower slopes of Dionysius. The streaks measured from twenty to forty kilometers in length, appeared absolutely straight, and could have been but a few hundred meters in breadth. They have not been seen in Cambridge. Occasionally we find streaks flowing towards Tycho, even in its immediate vicinity. A most striking instance of this occurred in Clavius, where, upon January 30, 1893, I saw two minute craterlets situated side by side, one sending its streak away from Tycho and the other directly towards it.

It is sometimes stated as proving the total absence of a lunar atmosphere that no clouds ever obscure the Moon's surface, but that all its features are at all times perfectly clear and distinct. This statement is certainly erroneous. The Moon's atmosphere

probably is full of clouds. Wherever we see a bright streak, there a few days after sunrise will be found a cloud, and it is chiefly their conspicuous presence, combined with the lack of shadows, that at the time of full Moon makes the lunar detail in certain regions so difficult to distinguish. In other regions, over the *maria*, for instance, and strikingly over the floor of Plato, where the bright streaks do not extend, far more detail can be seen at full Moon than during any other portion of the lunation.—*Annals of Harvard College Observatory, Vol. XXXII, Part II.*

NON-EUCLIDEAN GEOMETRY.

GEORGE BRUCE HALSTED.

For Popular Astronomy.

In writing of "The Wonderful Century," Alfred Russel Wallace says of all time before the seventeenth century: "Then, going backward, we can find nothing of the first rank except Euclid's wonderful system of geometry, perhaps the most remarkable mental product of the earliest civilizations."

But of late all men of science and intelligent teachers have been hearing more and more of non-Euclidean geometry, and are naturally anxious to know how these new doctrines are related to the traditional geometry which they were taught and perhaps now are teaching.

The new departure is absolutely epoch-making, but fortunately it has intensified admiration for that imperishable model, already in dim antiquity a classic, the immortal *Elements* of Euclid.

But without assumptions nothing can be proved, and Euclid stated his assumptions with the most painstaking candor. He would have smiled at the suggestion that he could ever claim for his conclusions any other truth than perfect deduction from assumed hypotheses.

And so his system is forever safe. Each one of his axioms may turn out to be inconsistent with external reality; each of his fundamental assumptions may be replaced in our final explanation of the space in which we live and move; in reference to our space, all his theorems may be shown to be only approximations; and yet his work will remain a perfect piece of pure mathematics, the exact, eternal geometry of Euclidean space.

For two thousand years no one ever doubted the truth of any one of this set of axioms, far the most influential in the intellectual history of the world, put together by Euclid in Egypt,

but really owing nothing to the Egyptian race, nothing to the boasted lore of Egypt's priests.

The Papyrus of the Rhine, belonging to the British Museum, but given to the world by the erudition of a German Egyptologist, Eisenlohr, and a German historian of mathematics, M. Cantor, gives us more knowledge of the state of mathematics in ancient Egypt than all else previously accessible to the modern world. Its whole testimony confirms with overwhelming force the position that geometry as a science, strict and self-conscious deductive reasoning, was created by the subtle intellect of the same race whose bloom in art still overawes us in the Venus of Milo, the Apollo Belvidere, the Laocoön. But though for twenty centuries the truth of the axioms of the Greek geometer remained unquestioned, there was one of them of which the axiomatic character was doubted even from far antiquity. Elementary geometry was for two thousand years as stationary, as fixed, as peculiarly Greek as the Parthenon. But among Euclid's assumptions is one differing from the others in prolixity, whose place fluctuates in the manuscripts.

Peyrard, on the authority of the Vatican MS., puts it among the postulates, and it is often called the parallel-postulate. Heiberg, whose edition of the Greek text is the latest and best (Leipzig, 1883-1888), gives it as the fifth postulate.

James Williamson, who published the closest translation of Euclid we have in English, indicating, by the use of italics, the words not in the original, gives this assumption as eleventh among the Common Notions.

Bolyai speaks of it as Euclid's Axiom XI.

Todhunter has it as twelfth of the Axioms.

Clavius (1574) gives it as Axiom 13.

The Harpur Euclid separates it by forty-eight pages from the other axioms.

It is not used in the first twenty-eight propositions of Euclid. Moreover, when at length used, it appears as the inverse of a proposition already demonstrated, the seventeenth, and is only needed to prove the inverse of another proposition already demonstrated, the twenty-seventh.

Geminos of Rhodes (about 70 B. C.) speaks of it as needing proof. The astronomer Ptolemy (A. D. 87-165) tried his hand at proving it.

The great Lambert expressly says that Proklas demanded a proof of this assumption because when inverted it is demonstrable.

The Arab Nasir-Eddim (1201-1274) tried to demonstrate it.

No one had a doubt of the necessary external reality and exact applicability of the assumption. Until the present century the Euclidean geometry was supposed to be the only possible form of space-science; that is, the space analyzed in Euclid's axioms was supposed to be the only non-contradictory sort of space.

But could not this assumption be deduced from the other assumptions and the twenty-eight propositions already proved by Euclid without it? Euclid demonstrated things more axiomatic by far. He proves what every dog knows, that any two sides of a triangle are together greater than the third.

Yet after he has finished his demonstration, that straight lines making with a transversal equal alternate angles are parallel, in order to prove the inverse, that parallels cut by a transversal make equal alternate angles, he brings in the unwieldy assumption thus translated by Williamson (Oxford, 1781).

"II. And if a straight line meeting two straight lines make those angles which are inward and upon the same side of it less than two right angles, the two straight lines being produced indefinitely will meet each other on the side where the angles are less than two right angles."

As Staeckel says, "it requires a certain courage to declare such a requirement, alongside the other exceedingly simple assumptions and postulates."

In the brilliant new light given by Bolyai and Lobachevski, we now see that Euclid understood the crucial character of the question of parallels.

There are now for us no better proofs of the depth and systematic coherence of Euclid's masterpiece than the very things which, their cause unappreciated, seemed the most noticeable blots on his work.

Sir Henry Savile, in his Praelectiones on Euclid, Oxford, 1621, p. 140, says: "In pulcherrimo Geometriae corpore duo sunt naevi, duae labes . . . " etc., and these two blemishes are the theory of parallels and the doctrine of proportion; the very points in the Elements which now arouse our wondering admiration.

But down to our very nineteenth century an ever renewing stream of mathematicians tried to wash away the first of these supposed stains from the most beauteous body of Geometry.

The attempts may be divided into three classes: First, those in which is taken a new definition of parallels. Second, those in which is taken a new axiom different from Euclid's. Third, the

largest and most desperate class of attempts, namely, those which strive to deduce the theory of parallels from reasonings about the nature of the straight line and plane angle. Hundreds of mathematicians tried at this. All failed. That eminent man, Legendre, was trying at this, and continually failing at it, throughout his very long life. Thus the experience of two thousand years went to show that here some assumption was indispensable. Every species of effort was made to avoid or elude it, but without success. From a letter of Gauss we see that in 1799 he was still trying to prove that Euclid's is the only non-contradictory system of geometry, and that it is the system regnant in the external space of our physical experience. The first is false; the second can never be proven.

Yet even in 1831 the acute logician, De Morgan, accepted and reproduced a wholly fallacious proof of Euclid's assumption, recently republished, Chicago, 1898.

A like pseudo-proof published in *Crelle's Journal* (1834) deceived even our well known Professor W. W. Johnson, who translated and published it in the *Analyst* (Vol. III, 1876, p. 103), saying, "this demonstration seems to have been generally overlooked by writers of geometrical text-books, though apparently exactly what was needed to put the theory upon a perfectly sound basis."

The most interesting and perhaps the most extended of such attempted proofs was by the Italian Jesuit Saccheri, born the fifth of September, 1667, who joined the Society of Jesus at Genoa, on the twenty fourth of March, 1685. He became teacher of grammar in the Jesuit "Collegio di Brera," where the teacher of mathematics was Tommaso Ceva, a brother of the well known mathematician, Giovanni Ceva (1648-1737), who published in 1678 at Milan a work containing the theorem now known by his name.

Saccheri was in close scientific communion with both brothers and received his inspiration from them. He used Ceva's ingenious methods in his first published work, 1693, solutions of six geometric problems proposed by Count Roger Ventimiglia. His attempt at proving the parallel-postulate in his last work, "Euclid vindicated from every fleck," which received the "Imprimatur" of the Inquisition the thirteenth of July, 1733, that of the Provincial of the Jesuits the sixteenth of August, 1733. Saccheri died the twenty-fifth of October, 1733.

All preceding attempts were alike in trying to give a direct positive proof of the postulate; all were alike in the assumption

open or hidden, conscious or unconscious, of an equivalent postulate.

Saccheri tries a wholly new way, and thus his book marks an epoch. He never doubted the absolute necessary truth of Euclid's postulate, and so he thinks that the two alternatives, possible if it be taken as not true, must each lead to some contradiction, to some absurdity. He tries the reductio ad absurdum. Ninety years later, 1823, Bolyai Janos reached the astounding conviction that these alternatives lead not to any contradiction but to the "science absolute of space," a generalization of Euclid's universe. In a letter dated the third of November, 1823, written in the Magyar language, and fortunately preserved for us at Maros Vásárhely in Hungary, Bolyai János writes to his father, Bolyai Farkas: "I have discovered such magnificent things that I am myself astonished at them. It would be damage eternal if they were lost. When you see them, my father, you youself will acknowledge it. Now I cannot say more, only so much: *that from nothing I have created another wholly new world.*"

Suppose we take a few steps into this new universe on the path which opened before Saccheri without his every suspecting whither it led.

1. If two points determine a line it is called a straight.

2. If two straights make with a transversal equal alternate angles, they have a common perpendicular.

3. A piece of a straight is called a sect.

4. If two equal coplanar sects are erected perpendicular to a straight, if they do not meet, then the sect joining their extremities makes equal angles with them and is bisected by a perpendicular erected midway between their feet: [Proved by folding the figure over, along the third perpendicular].

5. Considering figures where the right angles made by the equal perpendiculars may be said to be not alternate, and where no two perpendiculars to the same straight meet, the equal angles made with the joining sect at the extremities of the two equal perpendicular are either right angles, acute angles, or obtuse angles. Distinguish the three cases as hypothesis of right, hypothesis of acute, hypothesis of obtuse.

6. According to these three hypotheses respectively, the join of the extremities of the equal perpendiculars is equal to, greater than, or less than the join of their feet. [Saccheri, Prop. III. Translated by Halsted in the *American Mathematical Monthly*].

7. Inversely, according as the join of the extremities is equal

to, or less than, or greater than the join of the feet, the equal angles will be right, or obtuse or acute. [S. P. IV].

8. Corollary. In every quadrilateral containing three right angles and one obtuse, or acute, the sides adjacent to this oblique angle are less than the opposite sides, if this angle is obtuse, but greater if it is acute.

9. The hypothesis of right, if even in a single case it is true, always in every case it alone is true. [S. P. V.].

10. Assuming the principle of continuity, and referring only to figures where no two perpendiculars to the same straight meet; The hypothesis of obtuse, if even in a single case it is true, always in every case it alone is true. [S. P. VI.].

11. With like limitation; The hypothesis of acute, if even in a single case it is true, always in every case it alone is true. [S. P. VII.]

12. The sum of the angles of a retilineal triangle is a straight angle in the hypothesis of right, is greater than a straight angle in the hypothesis of obtuse, is less than a straight angle in the hypothesis of acute. [S. P. IX].

13. The excess of a triangle is the excess of the sum of its angles over a straight angle. The deficiency of a triangle is what its angle sum lacks of being a straight angle.

14. Two triangles having the same excess or deficiency are equivalent.

15. Even with the assumption that two straights cannot intersect in two points, the three hypotheses give rise to three perfect systems of geometry, the hypothesis of right to Euclid, the hypothesis of acute to Bolyai-Lobachevski, the hypothesis of obtuse to Riemann.

16. In the hypothesis of acute the straight is infinite. Two coplanar straights perpendicular to a third diverge on either side of their common perpendicular. The angle-sum of any rectilineal triangle is less than a straight angle.

17. In Euclid and Bolyai, parallels are straights on a common point at infinity.

In Bolyai, from any point P drop PC a perpendicular to a given straight AB.

If D move off indefinitely on the ray CB the sect PD will approach as limit PF copunctal with AB at infinity. PF is said to be at P the parallel to AB toward B.

PF makes with *PC* an angle *CPF* which is called the angle of parallelism for the perpendicular *PC*. It is less than a right angle by an amount which is the limit of the deficiency of the triangle *PCD*. On the other side of *PC* an equal angle of parallelism gives us the parallel at *P* to *BA* toward *A*.

Thus at any point there are two parallels to a straight.

A straight has two distinct separate points at infinity.

Straights through *P* which make with *PC* an angle greater than the angle of parallelism and less than its supplement do not meet the straight *AB* at all, not even at infinity.

18. A straight maintains its parallelism at all its points. [Lobachèvski, Geometrical Researches on the Theory of Parallels, Translated by Halsted, § 17].

19. If one straight is parallel to a second, the second is parallel to the first. [L. § 18.].

20. Two straights parallel to a third toward the same part are parallel to each other. [L. § 25].

21. Parallels continually approach each other. [L § 24].

22. The perpendiculars erected at the middle points of the sides of a triangle are all parallel if two are parallel. [L. § 30.]

23. If the foot of a perpendicular slides on a straight, its extremity describes a curve called an equi-distant curve or an equidistantial.

An equidistantial will slide on its trace.

24. A circle with infinite radius is not a straight but a curve, called the boundary curve, which is a plane curve for which all perpendiculars erected at the mid-points of chords are parallel. [L. § 31]. It is an equi-distantial whose base line is infinitely removed.

Circles, boundary-curves, equi-distantials cut at right angles a system of copunctal straights, of parallel straights, of perpendiculars to a straight, respectively.

Three points determine one of these curves; that is through any three points not co-straight will pass either a circle, a boundary-curve, or an equi-distantial, and only one such curve.

Any triangle may be inscribed in one and only one of these curves.

25. Boundary-surface we call that surface generated by the revolution of a boundary-curve about one of its axes.

Principal plane we call each plane passed through an axis of the boundary-surface.

Every principal plane cuts the boundary-surface in a boundary-curve.

Any other plane cuts the boundary-surface in a circle.

Boundary-triangles whose sides are arcs of the boundary-curve on the boundary-surface have the same interdependence of angles and sides and the same angle sum as rectilineal triangles in Euclid.

Geometry on the boundary-surface is the same as the ordinary Euclidean plane geometry. [L. § 34].

26. Triangles on an equidistant surface are similar to their projections on the base plane; that is, they have the same angles and their sides are proportional.

27. In the hypothesis of obtuse, a straight is of finite size, and returns into itself.

This size is the same for all straights. Any two straights can be made to coincide.

Two straights always intersect.

Two straights perpendicular to a third intersect at a point half a straight from the third either way.

28. A straight in the hypothesis of obtuse does not divide the plane into hemiplanes.

Starting from the point of intersection of two straights and passing along one of them over a certain finite sect, we come again to the intersection without having crossed the other straight.

This sect is the whole straight, and so a straight has not really two sides.

There is one point through which pass all the coplanar perpendiculars to a given straight. It is called the pole of that straight, and the straight is its polar.

A pole is half a straight from its polar. A polar is the locus of coplanar points half a straight from its pole. Therefore if the pole of one straight lies on another straight, the pole of this second straight is on the first straight.

The cross of two straights is the pole of the join of their poles.

The equidistantial is a circle with center at the pole of its basal straight.

Three straights each perpendicular to the other two form a tri-rectangular triangle. It is self-polar, each vertex being the pole of the opposite side.

29. In the hypothesis of obtuse, any two straights enclose a plane figure, a digon.

Two digons are congruent if their angles are equal.

30. In the hypothesis of obtuse, all perpendiculars to a plane meet at a point, the pole of the plane. It is the center of a system of spheres of which the plane is a limiting form when the radius becomes equal to half a straight.

Figures on a plane can be projected from similar figures on any sphere which has the pole of the plane for center. They have equal angles and corresponding sides in a constant ratio depending only on the radius of the sphere.

Geometry on a plane is therefore like two-dimensional spherics, but the plane corresponds to only a hemisphere.

The plane is unbounded but not infinite. It is finite in extent. The universe is unbounded but not infinite. It is finite in extent, or content, or volume.

Now of these three possible geometries of uniform space, Euclid's has the unexpected disadvantage that it can never be proved to be the system actual in our external physical world. To establish Euclid, it would be necessary to show that the angle-sum of a triangle is *exactly* a straight angle; and no measurements can ever reach exactitude.

To prove one of the others, we have only to show that the sum of the angles of some triangle is *less than*, or *greater than* a straight angle, which may conceivably be done even by inexact measurements.

What changes ought to be made in teaching elementary geometry in consequence of these later discoveries and the principles of the non-Euclidean geometries?

We are given a new criterion for questions of method, of exposition. For example, surface spherics attains a new importance.

When properly founded and expounded, pure spherics, two-dimensional spherics, while giving all the old results and laying the foundation for spherical trigonometry, gives also a picture of the planimetric part of Riemann's geometry, and becomes a touchstone for detecting the fallacies and assumptions in the many pseudo-proofs accepted in the past, such as attempts to found parallelism on direction, attempts to prove all right angles equal, etc.

As another example, we see a new stress laid on the incalculable advantages, educational and scientific, of Euclid's procedure in deducing from three assumed constructions every other construction before he uses it in any demonstration. The glib method of *supposed* solutions to all desired problems, of hypothetical constructions, is now seen in its deformity and danger.

Euclid says, under the heading "Postulates:"

"I. It is assumed, that a straight line may be drawn from any one point to any other point.

"II. And that a terminated straight line [a sect] may be produced in a straight line continually.

"III. And that a circle may be described with any center and radius."

From these Euclid rigidly deduces every problem of construction he wishes to use. Says Helmholtz: "In drawing any subsidiary line for the sake of his demonstration, the well-trained geometer asks always if it is possible to draw such a line. It is notorious that problems of construction play an essential part in the system of geometry. At first sight these appear to be practical operations, introduced for the training of learners; but in reality they have the force of existential propositions. They declare that points, straight lines, or circles, such as the problem requires to be constructed, are possible under all conditions, or they determine any exceptions that there may be."

Euclid's first three propositions are problems.

The most popular American geometry, Wentworth's, (1899), puts Euclid's two first postulates on page 8, and the third postulate a whole book later, and then never has a single problem of construction until page 112, where he says: "Hitherto we have supposed the figures constructed."

Meantime, on page 88, he gives as a "theorem:" "Through three points not in a straight line, one circumference, and only one, can be drawn.

He gives as his "Proof. Draw the chords AB and BC. At the middle points of AB and BC suppose perpendiculars erected. These perpendiculars will intersect at some point O, since AB and BC are not in the same straight line."

Now the tremendous existential import of the problem, to draw a circle through three non-costraight points, will be recognized when I say that in general it is not possible. In the Lobachevski geometry not every triangle has its vertices concyclic. Granting that every three points must be costraight or concyclic, we prove the parallel-postulate.

Of the possible geometries we cannot say *a priori* which shall be that of our actual space, the space in which we move.

The hereditary geometry, the Euclidean, is underivable from real experience alone, and can never be proved by experience. Euclidean space is, at least in part, a creation of the human mind. Its adequacy as a subjective form for experience has not yet been disproved. It can never be proved.

The realities which with the aid of our subjective space form we understand under motion and position, may, with the coming of more accurate experience, refuse to fit in that form.

Our mathematical reason may decide that they would be fitted better by a non-Euclidean space form.

Comparative geometry finally overthrows that superficial method which pretends to found a logically sound exposition of geometry on "direction," undefined.

For more than 20 years Wentworth gave his definition "A straight line is a line which has the same direction throughout its whole extent." (1877, Def. 8. 1886, p. 4; 1888, § 17).

At last he discards his aged error, and takes the definition of non-Euclidean geometry, "a straight is the line determined by two points" (1899, §§ 36 and 46).

Though the Bolyai and the Riemann geometries are founded on the straight, yet to say in them of two straights that they have the same direction has no ordinary meaning, since in Riemann every two straights cross and inclose a space, while in Bolyai every two parallels continually approach each other.

So as to direction, Wentworth has reformed, after 20 years in the land of nod.

But he still says, 1899, § 49, "A straight line is the shortest line that can be drawn from one point to another."

Now a relation of equality or inequality between two magnitudes must have some foundation, and be capable of some intelligible test. In the traditional geometry the foundation of all proof by Euclid's method consists in establishing the congruence of magnitudes.

To make the congruence evident, the geometrical figures are supposed to be applied to one another, of course without changing their form and dimensions. But since no part of a curve can be congruent to any piece of a straight, so, for example, no part of a circle can be equivalent to any sect from the definition of equivalent magnitudes as those which can be cut into pieces congruent in pairs.

In any comparison of size by congruence, we must be able to place one of the magnitudes or portions of it in complete or partial coincidence with the other. No such direct comparison can be instituted between a straight and a line no piece of which is straight. Thus the whole of Euclid's *Elements* fails utterly to institute or prove any relation as regards size between a sect and an arc joining the same two points. The operation of measurement we cannot effect, rigorously speaking, either for curves or for curved surfaces, since the unit for length is a sect, and the unit for area, the square on that sect. In fact, however little may be the parts of a curve, they do not cease to be curved, and consequently they cannot be compared directly with a sect; just as parts of a curved surface are not directly comparable with portions of a plane.

We cannot even affirm that any ratio exists between a circle and its diameter until after we have made some extra-Euclidean and post-Euclidean assumption at least equivalent to the following:

No minor arc is less than its chord; and no arc is greater than the sum of the tangents at its extremities. If the curve be other than a circle we assume that on it one can always take two points so near that the arc between these points is not less than its chord, nor greater than the broken line formed by the two tangents touching its extremities. Some such assumption is, in fact, necessary, but it destroys by itself the primitive idea of measuring curves with straights.

Dahamel gives the assumption the following form: The length of a curve shall be the limit toward which the length of a broken line made up of consecutive chords of that curve approaches, when the number of chords is increased in such a manner that the chords all approach zero as limit.

Thus the evaluation of the length of a curve represents not at all an attempt at rectification strictly; but it has for aim the finding of a limit to which another magnitude would approach.

In geometry one proves that as the subdivisions are increased and the sides tend toward the limit zero, the perimeter of the polygon inscribed in a circle increases, circumscribed decreases, toward the same limit, which then is assumed for the magnitude of the circle.

Therefore when Phillips and Fisher, of Yale, give as their definition of a straight (1898, p. 4, § 7. Def.) "A *straight line* is a line which is the shortest path between any two of its points," they pass through and beyond Euclid's *Elements* to give us his simplest element; they institute a comparison not only with circular arcs, but also with all curves known and unknown; they presuppose a foreknowledge of all lines in a definition of the simplest line. Is it still needful to say this is grossly bad logic, bad pedagogy, bad mathematics.

The scientific doctrine of evolution postulates a world independent of man, and teaches the outcome of man from lower forms of life in accordance with wholly natural causes. In this world of evolution experience is a teacher, but man is a creator, and the mighty examiner is death.

The puppy born blind must still be able, guided by the sense of smell, to superimpose his mouth upon a source of nourishment. The little chick, responding to the stimulus of a small bright object, must be able to bring his beak into contact with the object

so as to grasp and then swallow it. The springing goat that misjudges an abyss is lost.

So too with man. His ideas must in some way correspond to this independent world, or death passes upon him an adverse judgment.

But it is of the very essence of the doctrine of evolution that man's metric knowledge of this independent world, having come by gradual betterment and through imperfect instruments, for example the eye, cannot be absolute and exact.

The results of any observations are always with certain definite limitations as to exactitude and under particular conditions.

Man the creator replaces these results by assumptions presumed to have absolute precision and generality, such as, for example, the so-called axioms of Euclid.

If two natural hard objects, susceptible of high polish, be ground together, their surfaces in contact may be so smoothed as to fit closely together and slide one on the other without separating. If now a third surface be ground alternately against each of these two smooth surfaces until it accurately fits both, then we say that each of the three surfaces is approximately plane, is a piece of a plane.

If one such plane be made to cut through another, we say the common line where they cross is approximately a straight.

The perfect, the ideal plane is a human creation under which we seize the imperfect data of experience.

If three approximate planes on real objects be made to cut through a fourth approximate plane, then the three approximate straights are formed on this fourth plane, and in general they are found to intersect, and the figure they make we may call an approximate triangle.

Such triangles may vary greatly in shape. But no matter what the shape, if we cut off the six ends of any two such, and place them side by side on a plane with their vertices at the same point, the six are found with a high degree of approximation just to fill up the plane about the point. If the whole angular magnitude about any point in a plane be called a perigon, then we may say that the six angles of any two approximate triangles are found to be together approximately a perigon.

Now does the exactness of this approximation to a perigon depend only on the straightness of the sides of the original two triangles, or also upon their size?

If we know with absolute certitude that the size of the trianangles has nothing to do with it, then we know something that

we have no right to know according to the doctrine of evolution, something impossible for us ever to have learned evolutionally.

Yet before the epoch-making ideas of Bolyai János and Lobachevski, everyone supposed we were perfectly sure that the angle-sum of an actual approximate triangle approached a straight angle with an exactness dependent only on the straightness of the sides and not at all on the size of the triangle.

But if in the mechanics of the world independent of man we were absolutely certain that all therein is Euclidean and only Euclidean, then Darwinism would be disproved by the reductio ad absurdum.

All our measurements are finite and approximate only.

The mechanics of actual bodies in what Cayley called the external space of our experience, might conceivably be shown by merely approximate measurements to be non-Euclidean, just as a body might be shown to weigh more than two grams or less than two grams, though it never could be shown to weigh precisely, absolutely two grams.

The outcome of the non-Euclidean geometry is a new freedom to explain and understand our universe and ourselves.

MERCURY AS A NAKED EYE OBJECT.

W. F. DENNING.

Rarely visible, and always difficult to observe satisfactorily in a telescope, this planet is yet a most attractive object to the unaided eye. Not receding to a greater distance than 28° from the Sun, he is, however, never above the horizon in England for a longer period than two and a quarter hours before sunrise, or for a similar interval after sunset. When an evening star in the spring months or a morning star in the autumn season, he may often be caught and watched for an hour or so, shining with a sparkling, rosy lustre, and presenting much the same aspect as a fixed star.

To secure a view of Mercury forms one of the earliest and greatest ambitions of the amateur astronomer. Among his first books there will sure to be a copy of Mitchell's "Orbs of Heaven," or Dick's "Celestial Scenery," and on reading the statement that Copernicus never succeeded in seeing Mercury, he resolves that he will do his best to catch a glimpse of this elusive little "Messenger of the Gods." After some vain attempts he finally succeeds,

and it is not too much to say that the spectacle sometimes excites and gratifies the observer more than any other subsequent event in his astronomical career. Who is there among us who does not remember the thrill of pleasure incited by the first detection of this fugitive orb, and the conscious pride with which we realized that we had commenced our celestial work by achieving a feat which had been denied to the greatest astronomer of the sixteenth century?

But, as a matter of fact, there seems to be considerable doubt whether Copernicus ever really complained of failure to see Mercury. There is evidence to show that he never expressed himself in the manner quoted in many of our popular text-books. There may, it is true, have been some ground for the statement, but it is well known that a biographer has only to introduce a special incident of the kind alluded to, or to unduly color some expression, and whether on doubtful evidence or not, it is liable to be copied and recopied by subsequent writers without any investigation until it becomes generally accepted as a fact. But admitting for the moment that Copernicus really failed to discern Mercury, he seems to have had very good reason for it. His residence was at Thorn, in Prussia, and through the valley near ran the River Vistula, over which were frequent fogs which obliterated objects near the horizon.

This tradition about Copernicus and Mercury has certainly, however, enhanced the interest with which the planet is regarded as a naked-eye object. The beautiful white lustre of Venus—incomparably brighter than the aspect of Mercury—the stronger and steadier, yellowish light of Jupiter, or the conspicuous ruddy hue of Mars may present a more striking appearance in the sky than the twilight-veiled splendor of Mercury, but there is something about the sparkling lustre of the latter orb, hovering fugitively on the brow of the horizon, which forms an attraction peculiarly its own.

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On reference to my note-book I find that I obtained naked-eye views of Mercury on 102 occasions between February, 1868, and December, 1899. But the planet was very rarely looked for here at the morning apparitions, and not always at really favorable spring elongations. If an observer with good sight made it a point to secure as many unassisted eye observations of this object as possible, he might be successful on about twelve occasions in a year. In a finer climate than ours, the planet may, of course, be more frequently seen. I think that some disappointments in

regard to finding Mercury are due to the fact that observers scan the heavens at or after the time of maximum eastern elongations, instead of during a week or more preceding them. The phase and apparent brilliancy decrease rapidly at these periods. I have occasionally noticed Mercury as a very brilliant object about ten or twelve evenings before his greatest elongation, while at the date of his elongation he has appeared quite faint, and a few evenings later, become practically invisible, though above the horizon for about two hours after sunset.

My observations in various years have led me to the following conclusions regarding the visibility of the planet at the evening apparitions:—

(1) The greatest brightness of the planet is attained ten or twelve days prior to his greatest elongation.

(2) In February and March the planet may sometimes be caught twenty minutes after sunset, in April thirty minutes after sunset, and in May forty minutes after sunset. The stronger twilight towards midsummer occasions the difference.

(3) The duration of his visibility to the naked eye is about 1^h 40^m in March, 1^h 30^m in April, and 1^h 20^m in May. On a very exceptional occasion it is possible these limits may be exceeded.

(4) The planet is a conspicuous object, and certainly much brighter than a 1st mag. star. In February, 1868, I considered that his lustre vied with that of Jupiter, then only 2° or 3° distant. In November, 1882, he appeared brighter than Sirius. In 1876 he looked more striking than Mars, then 13° distant, but the latter planet was faint and at a considerable distance from the Earth.

The greatest number of naked eye observations of Mercury at the same elongation was obtained at Bristol in spring of 1876, when the planet was seen on thirteen different evenings. When Venus is near Mercury at a favorable time, she affords an excellent guide to the identification of the latter. But errors have often been induced, and either Venus or Jupiter has been mistaken for Mercury on many occasions. In April, 1898, Venus was near Mercury, and some people, including a few regular astronomical observers, readily saw Venus and believed (and still ardently believe) that they were looking at Mercury.

The albedo, or reflecting capacity of the planet, is rated exceedingly low, being only 0.11, whereas Mars is 0.27, Saturn 0.50, and Venus and Jupiter 0.62. This is remarkable when we consider the occasional striking brightness of the small planet in a region of the sky full of strong twilight. By telescopic comparisons of

the disc of Mercury with other planets, it is, however, easily seen that the former is relatively feeble in brilliancy. On May 12, 1890, I viewed Mercury and Venus in the same field of view of a 10-inch reflector, and remarked that the brilliant silvery light of Venus contrasted strongly with the much duller hue of Mercury. The probability is that the latter object is provided with a much thinner atmosphere than that which envelopes his sister planet. There are undoubted markings visible on Mercury, but they are nothing like the peculiar representations of them which have been published in the last few years. The extreme difficulty of obtaining satisfactory views of the planet furnishes the principal reason why his rotation period still awaits accurate determination.

Nature, March 1, 1900.

THE LUNAR ATMOSPHERE.

In the light of the observations already described, we now seem justified in concluding that the Moon is surrounded by an atmosphere. Indeed, nearly all the volcanic phenomena described in this volume involve the escape of imprisoned gases. The visual and photographic observations of the occultation of Jupiter, described upon pages 82 and 83, indicate that upon the sunlit side of the Moon this atmosphere is filled to a height of about six kilometers, or four miles, with some absorbing medium, which is absent upon the dark side. These same observations upon Jupiter show that the gaseous tension at the lunar surface is so slight that the refraction 2 *H* is less than 0″.5. Since it was necessary to give very short exposures, the photographs could not be taken upon a larger scale than 5″ to a millimeter. The 13 inch telescope was employed, but with a larger aperture and consequently more light it would be possible to take the negatives upon a larger scale, and thus obtain more accurate results.

Upon the night of October 13, 1897, the moon, which was full upon October 10, occulted Alcyone in the Pleiades. As seen from Cambridge the star disappeared behind the bright limb at a point near Anaximander, or about twenty degrees to the east of the north pole. Time of disappearance $12^h\ 35^m$ G. M. T. The micrometer thread had previously been set to coincide with the star and its brightest companion *p* (Bessel). P. A. 289°, Distance 117″. The stars were carefully watched as they approached the limb of the Moon, but not the slightest deviation

in the position angle due to lunar refraction could be detected. At the time of the disappearance of Alcyone, p was still about 15″ from the Moon's limb. Alcyone reappeared from behind the dark limb at $13^h\ 15^m$. The micrometer had not been moved in the mean time, and the coincidence of its thread with the two stars was still found to be exact. The position angle of the thread was now changed by 0°.2. The effect upon the stars was quite marked, and it is very certain that any such displacement of the stars with regard to the thread would have been noticed. It was found that a change of 0°.1 would hardly have been seen, although it might have been. Since the distance between the stars was 117″, and the distance from the companion to the limb was 15″, the line joining the two stars was inclined at an angle of 7° to the surface of the Moon. A change of 0°.1 would therefore correspond to a displacement of Alcyone by lunar refraction of 0″.2.

Since the force of gravitation upon the surface of the Moon is but one-sixth as great as it is upon the Earth, the distance that we should have to ascend in the Moon's atmosphere to reduce the pressure by one-half is six times as great as the distance to which we must ascend above the surface of the Earth. In the case of the Earth this distance is three and one-half miles. To reduce the pressure of the lunar atmosphere by one-half we should have to ascend twenty-one miles, or about 17″ at the Moon's limb as seen from the Earth. Since at the disappearance of Alcyone p was still 15″ above the Moon's limb, the lunar refraction would displace it about one half as much as it did its primary. Therefore, if the change of position angle of the stars did not exceed 0°.1, the effect of the lunar refraction $2H$ could not exceed 0″.4 upon the bright side of the Moon.

This observation is really a repetition of one that I made at the Lowell Observatory in 1894, with the same result. The instrument employed in the present observation was the 15-inch equatorial, with a magnification of 550. The two fainter companions of Alcyone were both watched during the occultation, although they were seen with too much difficulty to obtain results of the highest accuracy had it been necessary to use them in the research. The photographic magnitudes of Alcyone and its three companions, according to these annals, Vol. XVIII, pp. 158, 159, are 3.42, 6.64, 8.14, and 8.69.

In the *Astrophysical Journal* for 1897, Vol. VI, p. 421, is given a list of twelve observations made between May 8 and October 3, 1897, by Professor Comstock, of occultations of stars ob-

served with a filar micrometer. All of his observations were of immersions on the dark side of the Moon. From them he concludes that upon the dark side the maximum permissible value of $2H$ is $0''.2$.

When observing occultations of stars with the filar micrometer, if the direction of the line joining the stars is nearly parallel to the occulting limb, their refraction should be measured, as in the above case, by means of the observed position angle. If the line joining the stars is nearly perpendicular to the limb, the refraction should be measured by determining the variation, if any, in the distance between the two stars. In either case the most accurate results can be obtained when the star is occulted near the north or south limb of the Moon. Under these circumstances we have more time in which to make the observation, and we have less trouble from the irregular running of the telescope, as the threads then lie in an approximately east and west direction. It is believed that greater accuracy would be obtained if one could measure the distance between two stars rather than if one were obliged to measure their position angle.

TABLE LXII.

LIST OF WIDE DOUBLE STARS AND CLUSTERS OCCULTED BY THE MOON.

No.	Object.	Magn.		P. A.	Dist.
				°	″
1	77 Piscium	5 9	3.8	82.7	32.8
2	ζ "	4.2	5.3	63.7	23.5
3	Pleiades				
4	χ Tauri	5 7	7.8	25.3	19 3
5	62 "	6 2	8	289.7	28.9
6	τ "	5	7.2	212.4	62.9
7	B. A. C. 1562	6	7	158.6	78.8
8	15 Geminorum	6	8	205 4	33.2
9	ν "	4 2	8	329.1	112.6
10	20 "	6	6 9	209.8	20
11	ζ "	4	7.2	351.6	93.6
12	Praesepe				
13	α Leonis	1.5	8.4	306.6	176.9
14	83 "	6.3	7 3	150.	29.6
15	τ "	5	7	169 6	94.8
16	B. A. C. 4134	5.9	6 4	196 3	20.1
17	α Librae	3	6		
18	23 Scorpii	4.1	7	336.8	40.8
19	σ "	3	8	271.2	20.4
20	6523 (M S) Sagitarii (cum.) Near B. A. C. 6088				
21	M 25 Sagittarii (cum.)				
22	σ Capricorni	5	8.3	177 2	55.8
23	β "	2.5	6	267.1	205.3
24	σ^2 "	6.3	6.8	239.9	21.8

In the preceding table, which has been deduced from "Celestial Objects for Common Telescopes," by T. W. Webb, is given a list of those double stars sometimes occulted by the Moon, whose distance exceeds 20″ and whose fainter component is brighter than magnitude 8.5. A few clusters of brilliant stars are also included.

Bearing in mind the most favorable conditions, that the stars should lie in a nearly north and south direction, that they should not be too far apart, and that the companion should not be too faint, we may select B. A. C. 4134 as decidedly the most favorable star for this investigation. After it follow, in the order of their desirability, 20 Geminorum, 23 Scorpii, 83 Leonis, and χ Tauri. The investigation is perhaps more likely to lead to a positive result if the disappearance takes place behind the bright limb of the Moon, as on account of the higher temperature an atmosphere is more likely to be found upon that side. The Moon should therefore be past the full. If the occultation is a grazing one, however, so that the relative motion of the Moon's limb and the star is slow, disappearances and reappearances can be observed with nearly equal accuracy, and the age of the Moon is unimportant.—*W. H. Pickering in H. C. O. Annals, Vol. XXXII, Part II.*

SPECTROSCOPIC NOTES.

The total eclipse of May 28 is now so near at hand that most of the subjects for study have been decided upon and many of the preparations for observation have been completed. The observations of recent eclipses will be repeated with care; and unless some substantial advance is scored this will at least have to be reckoned a less productive eclipse than the last three.

Probably the most extended study will be devoted to matters spectroscopic. The flash spectrum at the instants of beginning and end of totality will be further investigated, both photographically and visually, with various types of instrument, but especially with the prismatic camera. The question of the precise position of the corona line, made so interesting by the results of the last eclipse, will receive attention, the instrument being primarily the slit spectroscope. The distribution of 'coronium,' requires further study, and valuable work may be done with apparatus no more complex than a binocular with a direct vision prism before one of the glasses. The rotation of the corona, as shown by minute displacement of the corona line, can be studied better a year later, when totality will be much longer.

Among the opportunities for observation not distinctly spectroscopic are, large scale photographs of the corona with lenses of great focal length; with instruments of moderate size short exposure photographs to get the detail of the inner corona, and long exposure photographs to get the extension of the outer corona; measurement of the heat radiation of the corona; naked eye drawings of the corona, preferably of some limited portion of the corona only; telescopic ex-

amination of the corona, particularly in the neighborhood of the prominences; the polarization of the light of the corona; estimation of the total light of the corona.

Aside from the study of the Sun itself there are, a photographic search for intra-mercurial planets; determination of the brightness of Mercury with the whole of the disc illuminated, as at this eclipse Mercury will be nearly at superior conjunction; and a search, if the sky be dark enough, for a satellite of Mercury.

Intending observers who are not accustomed to thinking of eclipse matters would do well to consult some report of diversified eclipse observations, such as Sir Norman Lockyer's papers in *Nature*, or the Report of the Expedition of the British Astronomical Association to the Indian Eclipse; or some detailed explanation of possible observations, such as Professor Hale's "Eclipse Problems" (*Astrophysical Journal*, Jan. 1900), or Mr. Maunder's "Eclipse Suggestions" (*Journal British Astronomical Association*, Vol. 10, No. 4). But any observer in selecting his work is distressingly conscientious if he arranges a program which does not permit a plain view, at least momentary, of the corona.

Dr. Ambronn, in his elaborate *Handbuch der Instrumentenkunde*, devotes 44 pages out of 1264 to "*spectralapparate*;" vol. 2, chap. 14, pp. 725-768. The compilation and description of spectrum apparatus there given is avowedly less exhaustive than for other types of astronomical instruments. The first of the three sections of the chapter contains, briefly, a discussion of dispersion, achromatism, and the character of the spectrum lines; a description of prism systems employed, including compound and direct-vision prisms; and a statement of the theory of the cylindrical lens. In the second and longest section the various types of spectroscopes are enumerated and briefly described: The objective prism, the ocular direct-vision spectroscope; the slit spectroscope, with its various arrangements of slit and of index or micrometer, and in its variety of forms, as the star spectroscope of precision of the usual form, the half-prism spectroscope, and the grating spectroscope. The third section describes briefly the spectrograph, including the spectro-heliograph.

In a paper communicated to the Royal Astronomical Society (*Observatory*, January), Professor Barnard, describing his attempt to determine whether the faint nucleus of the ring nebula in Lyra is of the same constitution as the nebula, states that in the 40 inch telescope of the Yerkes Observatory the nucleus comes to focus 0.06 inches farther out than a star, while a planetary nebula comes to focus 0.26 inches farther out than a star.

Professor Campbell and Mr. Newall (*Observatory*, February) agree in finding for the period of the spectroscopic binary Capella 104 days. The two components, one of the solar type, the other of the type of Procyon, are found to be nearly equal in mass. The results for the velocity are from Professor Campbell's observations + 31 kilometres per second for the system, the component of the solar type ranging from + 4 to + 57 km.; from Mr. Newall's observations + 27 km. per second, the solar component ranging from 0 to + 54 km. per second.

A few pages later in the same number occurs the statement that several negatives of the spectrum of Capella, taken at the Washington Observatory, were rejected on account of the duplication of the spectrum lines, supposed to be due to some defect in the spectroscope.

Among the miscellaneous researches described in the *Harvard Observatory Annals*, Vol. 33, is an account of some observations on nebulae by Professors Pickering, Searle Upton and Wendell. Table III gives the general character of spectrum of about 100 nebulae, as determined by a direct vision prism used without a slit. The spectra are described as gaseous, continuous, gaseous with a faint continuous background, or chiefly continuous with a faint gaseous spectrum.

Professor Deslandres, using the great telescope of the Meudon Observatory with spectroscope specially designed for the measurement of motion of stars in the line of sight, finds the velocity of δ Orionis variable (*Comptes Rendus*, Feb. 12). The spectrum of this star shows little more than the lines of hydrogen and helium, which are broad and diffuse. The lines, further, seem to be variable in width, and are unsymmetrical. While the character of the lines precludes measures of great accuracy, Professor Deslandres regards the evidence for marked variation in velocity as adequate, and gives 1.92 days as a period agreeing with his observations.

Sir Norman Lockyer, *Proceedings of the Royal Society*, No. 422, adduces strong evidence for the presence in the Orion stars of the element silicon, or, to use his term, silicium. Several coincidences are found of lines of the element with lines of the star spectra. The lines of silicium may be divided into three sets, behaving differently as the electrical conditions are varied, and of these sets of lines one predominates in β Orionis, another in γ Orionis, and the third in ζ Orionis. The particular set of lines predominant in each star is that which is required by the author's previous conclusion that these stars are at different temperatures, β Orionis the coolest, γ Orionis intermediate, and ζ Orionis the hottest of the three.

PLANET NOTES FOR APRIL.

H. C. WILSON.

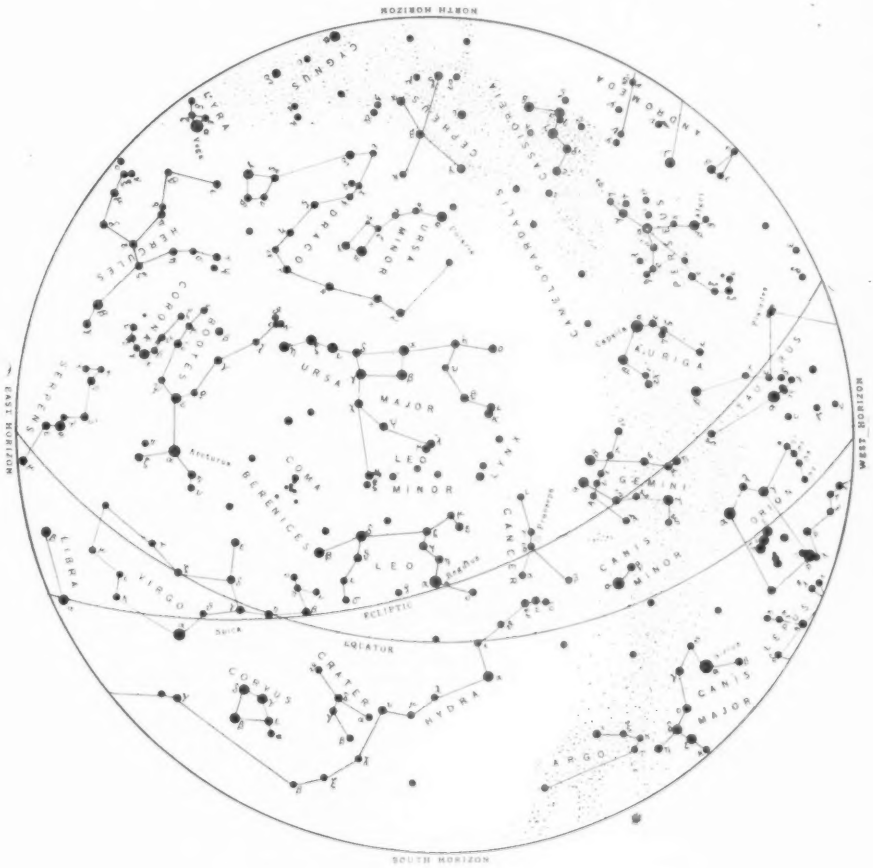
Mercury is now morning star, and will be visible to the naked eye during the latter half of the month. The planet will be at greatest elongation, west from the Sun 27° 19′, April 21. Mercury and Mars will be in conjunction April 3, but both will then be hidden in the glare of the Sun. Doubtless many were able to identify Mercury during the first two weeks of March, for it was quite conspicuous about a half hour after sunset.

Venus and the crescent moon will be in conjunction on the evening of April 2. In South America an occultation of Venus by the Moon will be visible at this time. Venus has been steadily moving northward as well as eastward, so that her position is very favorable for afternoon and evening observation. The planet will reach greatest elongation east from the Sun, 45° 35′, April 28. The phase of the planet is slightly gibbous, but will reach the quarter phase at the end of the month. Venus is at present approaching the Earth, so that her apparent brightness rapidly increases in spite of her diminishing phase.

Mars is morning star, but will not be in position for observation during this month.

Jupiter is approaching opposition, but is seen at such a low altitude in this country that observations will be of little value. The observers in South America

and Africa, on the other hand, will see the planet to the best advantage. Jupiter and the Moon will be in conjunction April 18. Jupiter rises at about midnight at the beginning of the month, and about 10 o'clock P. M. at the end of April.



THE CONSTELLATIONS AT 9 P. M., APRIL 1, 1900.

Saturn rises in the southeast about an hour and forty minutes later than Jupiter, and may be seen in the morning hours of darkness. Its position is equally unfavorable with that of Jupiter for us, and will be equally favorable for southern observers during the summer months. The Moon will be in conjunction with Saturn on the morning of April 20, at $9^h\ 39^m$ Washington mean time, and in the northwestern part of the United States an occultation may be witnessed.

Uranus is near Jupiter in the constellation Scorpio.

Neptune is at a good altitude in the western sky in the early evening, but is, of course, not visible to the naked eye. Neptune will be in conjunction with Venus on the morning of April 30, but the planets will be too far apart in declination, Neptune 4° 39′ north of Venus, for the one to be made use of in identifying the other.

The Moon.

		Phases.	Rises. (Central Standard Time at Northfield; Local Time 13m less.) h m	Sets. h m
April	6	First Quarter..... 10	52 A. M.	2 04 A. M.
	14	Full Moon..... 7	01 P. M.	5 25 "
	22	Last Quarter..... 1	22 A. M.	11 26 "
	28	New Moon..... 4	28 "	7 04 P. M.

Occultations Visible at Washington.

Date. 1900.		Star's Name.	Magnitude.	IMMERSION. Washington M. T. h m	Angle f'm N pt. °	EMERSION. Washington M. T. h m	Angle f'm N pt. °	Duration. h m
April	5	15 Geminorum	7.0	5 54	182	6 10	202	0 16
	11	e Leonis	5 3	17 08	94	17 59	304	0 51
	14	85 Virginis	6.5	16 40	181	16 57	208	0 17
	22	8 Aquarii	6.8	13 47	31	14 32	297	0 45

VARIABLE STARS.

J. A. PARKHURST.

Maxima and Minima of Long Period Variables.

MAXIMA, May 1900.

		Mag.	Day.
466	U Piscium	9.7	9
678	U Persei	7.7	29
782	R Arietis	8.3	18
869	R Fornacis	8.5	18
2258	V Aurigæ	9.2	8
2266	V Monocerotis	6.3	21
2376	S Lyncis	9	4
3493	R Leonis	6.0	5
4377	T Virginis	8.4	11
5511	RS Libræ	8	28
5593	W Libræ	9.8	26
5770	R Herculis	8.6	6
5931	S Ophiuchi	8.7	20
6100	RV Herculis	9.3	21
.....	... Aquilæ	9	28
7252	W Capricorni	10.6	22
7435	Y Aquarii	8.8	25
7455	U Capricorni	10.5	10
7571	V Capricorni	9	6
7659	T Capricorni	9 3	10
7733	Y Capricorni	10	6
7909	S Piscis Aust.	9.0	16
8153	R Lacertæ	8.8	13
8597	V Ceti	9.0	19

MINIMA, May 1900.

112	R Andromedæ	< 12.8	18
518	R Piscium	< 13	16
845	R Ceti	13 5	9
2539	R Canis Minoris	9 7	15
4521	R Virginis	9.8	17
4557	S Ursæ Majoris	10.8	18
5157	S Bootis	12.8	24
5237	R Bootis	11.8	20
6449	T Draconis	11.7	17
6871	V Lyræ	< 12	11
8591	V Cephei	6 9	6

MAXIMA, June 1900.

		Mag.	Day.
62	S Sculptoris	6.5	15
107	T Cassiopeæ	7.5	27
267	V Andromedæ	8 7	13
401	U Sculptoris	8.7	5
893	U Ceti	7.1	10
1116	X Ceti	9.3	30
2080	R Columbæ	7.9	17
2528	R Geminorum	7.2	10
2684	S Canis Minoris	7.6	6
2742	S Geminorum	8.5	15
2976	V Cancri	7 3	9

MINIMA, June, 1900.

.....	... Hydræ	8.5	1
3825	R Ursæ Majoris	7.1	9
4471	T Canum Ven.	8.7	13
4596	U Virginis	7.9	27
4896	T Centauri	5.9	7
5430	T Libræ	9.7	26
5501	S Serpentis	8.2	13
5583	X Libræ	9.7	28
5887	V Ophiuchi	7.3	3
6905	R Sagittarii	7.5	9
6943	T Sagittæ	8.5	16
7120	χ Cygni	5.3	13
7260	Z Aquilæ	8.9	26
7404	R Microscopii	8.0	5

MINIMA June, 1900.

7577	X Capricorni	10.0	3
7792	SS Cygni	8.5	11?
7896	V Pegasi	8.2	23
8068	S Lacertæ	7.7	21
8369	W Pegasi	8.1	17
.....	... Cassiopeæ	9.4	16

MINIMA, June 1900.

294	W Cassiopeæ	11.4	23
1623	T Camelopardalis	< 12	18
5494	S Libræ	< 13	26
5955	R Draconis	12.5	29
6512	T Herculis	10.7	18
7560	R Vulpeculæ	13	15

Minima of the Variable Stars of the Algol Type.

(Given to the nearest hour in Greenwich Time.)

1900 May.

U CEPHEI.

	d	h
May	4	13
	9	12
	14	12
	19	12
	24	11
	29	11

S VELORUM.

	d	h
May	11	17
	17	15
	23	14
	29	12

δ LIBRAE.

	d	h
May	1	20
	6	12
	8	20
	13	11
	15	19
	20	11
	22	19
	27	11
	29	18

U OPHIUCHI.

Every 10th min.

$P = 20.13^h$

	d	h
May	8	10
	16	20
	25	5

U CORONAE.

	d	h
May	2	14
	9	11
	12	22
	16	9
	19	20
	26	18

$+ 12°3557.$

	d	h
May	1	22
	2	20
	3	17
	4	14
	5	11
	9	22
	10	20
	11	17
	12	14
	13	12
	17	22
	18	20
	19	17
	20	14
	21	12
	25	22
	26	20
	27	17
	28	14
	29	12

Z HERCULIS.

Even epochs.

	d	h
May	2	16
	6	15
	10	15
	14	15
	18	14
	22	14
	26	14
	30	14

Odd epochs.

	d	h
May	4	15
	8	15
	12	15
	16	15
	20	14
	24	14
	28	14

$+ 45°3062.$

	d	h
May	3	17
	8	7
	12	21
	17	10
	22	0
	26	14
	31	4

W. DELPHINI.

	d	h
May	5	21
	10	16
	29	22

Y CYGNI.

Even epochs.

	d	h
May	3	16
	6	16
	9	16
	12	16
	15	16
	18	15
	21	15
	24	15
	27	15
	30	15

Odd epochs.

	d	h
May	1	18
	4	18
	7	18
	10	18
	13	18
	16	18
	19	18
	22	18
	25	18
	28	18

AUTHORITIES FOR THE EPHEMERIS.—These are the same as in the March number except that the times for Y Cygni and Z Herculis are also taken from Dr. Hartwig's paper in the *Vierteljahrsschritt*.

RECENT PUBLICATIONS ON VARIABLE STARS.—A number of important articles on this subject have appeared since the last month's notes, deserving more extended notice than can be given here. The following sketches will give some idea of their contents.

MIRA AND R LACERTAE.—In number 3622 of the *Nachrichten* Professor Deichmüller, of Bonn, gives his observations of Mira, covering the three maxima of 1887, 1888, and 1889, being the last three in the previous visibility-period, those of the four or five years following 1889 falling too near the Sun to admit of good determination. In connection with the date of each observation the author gives the comparison stars used, the brightness of the variable in steps, (the scale used agreeing closely with that of Argelander) and the conditions of seeing when at all doubtful. The results were as follows:

Dates of Observations.	No.	Date of Max.	Steps.
1887 Oct. 11 to 1888 Jan. 1	16	1887 Nov. 30	20
1888 Aug. 10 to 1888 Dec. 8	40	1888 Sept. 19.5	32
1889 July 31 to 1889 Oct. 5	10	1889 Aug. 20	27

Graphic representations of the light curves are given, showing at a glance the different types of maxima. The author calls attention to the importance of recognizing these different types, and of publishing the observations in sufficient detail to show them. A request is made that any unpublished observations be either published or communicated to him, as Mr. Guthnick, of Bonn, has undertaken a definitive reduction of this variable.

The same author gives his observations of R Lacertae in detail, covering the time from 1899 June 4 to Oct. 10, showing a principal maximum 1899 July 16.0, at 8.70 magnitude. The light curve is given, showing the rise from 12.0 magnitude to maximum and the decline to the same point. A chart of the field is given, and also a comparison with the elements derived from three well determined maxima in 1883, 1892 and 1899. The resulting elements of maximum are:

$$1899 \text{ July } 15.8 + 299^{d}.90 \text{ E.}$$

The corrections to these elements given by the three mentioned date of maxima, are only -0.2, $+0.7$, and -0.2 days, respectively.

These two papers are excellent examples of what reports of variables should be, and are recommended as models to be followed.

OBSERVATIONS OF VARIABLES BY ARGELANDER, SCHONFELD AND SCHMIDT.—Professor Pickering has done a valuable service to astronomy by printing, in Vol. XXXIII of the Harvard Annals, three series of observations of variables reduced with photometric values of the magnitudes of the comparison stars. The series made by Argelander includes about 4000 hitherto unpublished comparisons made between 1869 and 1871. These results are combined with his earlier work, done 1838 and 1867 and published in volume VII of the Bonn Beobachtunen. During this interval from 1838 to 1871, 16 long period variables were followed, making perhaps the most important series in existence.

Of scarcely less importance are the observations made by Schönfeld between 1853 and 1859, relating to 32 variables of long period. The relative brightness of the comparison stars used was determined with great care by Schönfeld, and a comparison with the photometric magnitude gives as the value of one step or "grade," 0.09 magnitude. The corresponding value in Argelander's work was 0.14 magnitude.

The work of Schmidt at Athens was done between 1845 and 1879, and is remarkable for the continuity of measures which the climate of Athens enabled him to secure. Of the many thousand of operations, only those are published

which relate to his long period variables, thirteen in number. It should be a warning to modern observers that of eighty-two comparison stars for these variables, Schmidt only left data for identifying thirty-four, and many of these are uncertain. The value of a "grade" expressed in photometric magnitudes is found to be 0.22. More than 7500 observations of the thirteen variables by this industrious observer are published.

The value of these old observations increases with age, since by combining them with modern comparisons our knowledge of the variations of the stars becomes more complete and accurate from year to year. Their reduction with photometric values of the comparison stars insures homogenity in the results, and the convenient form in which Professor Pickering has published them add greatly to their value.

ANOTHER NEW VARIABLE.—In No. 3618 of the *Nachrichten*, Rev. Thomas D. Anderson announces the discovery of a variable, not in the *DM*, in the place—

R. A. $17^h\ 55^m.6$, Dec. $+\ 54°\ 51'$ (1855)

Its observed magnitudes were—

1899 Nov. 14,	9.4
1900 Jan. 7,	10.2
26,	10.4

He gives the approximate places of the following comparison stars, which are not in the *DM*:

	Mag.	1855		
a	9.6	$17^h\ 53^m.9$,	$+\ 54°$	$51'$
b	9 8	17 53 .9,	54	44
c	10.6	17 54 .1,	54	37
d	10.3	17 55 .3,	54	41

The variable is nearly in line between the following two *DM*. stars—

$+\ 54°1925$	7.0^m	17^h	$54^m\ 39.10^s$	$+\ 54°$	$40'$	$49''$
$+\ 55\ 2003$	9.0	17	56 58 2,	55	2	39

The magnitudes and positions of the two *DM* stars are taken from the Gesellschaft Catalogues.

On the evenings of March 2 and 6 a 10.5 magnitude star was found near the indicated place and its position found from the two *DM* stars proved to be

R. A. 17^h	55^m	35.7^s,	Dec. $+\ 54°$	$50'$	$8''$, (1855)
	56	28.6,		49	52, (1900)

There is a 12th magnitude star $2^s.5$ following and $1'\ 17''$ south. If the stars given are platted it will be easy to recognize the field, which lies 2° south and ½° following the 4th magnitude star ξ Draconis.

THE NEW CERASKI ALGOL-TYPE VARIABLE.—In justice to the writer it should be stated that he did not see the proof of the chart or note concerning this star, given on pages 158 and 159 of the March number, or some of the printed errors would have been corrected.

COMET AND ASTEROID NOTES.

Photographic Search for Comet 1892, (Barnard).—During the summer and autumn of 1899, from July 10 to October 14, Dr. Schwassmann, at Heidelberg, Germany, executed a quite extensive photographic search for this comet, which though very faint visually was discovered by Barnard in 1892 upon a photograph of the Milky Way. Dr. Schwassmann took 25 photographs

on 19 nights, with average exposures of about two hours, selecting the field of the photograph by means of an ephemeris computed by M. Coniel, published in the *Bulletin Astronomique*. The plates each covered an area of about 100 square degrees. No trace of the comet was found, while stars of the 13th magnitude and fainter were photographed. Dr. Wolf in speaking of this remarks the few comets which are found, considering the large number of wide field photographs which are now being taken for the purpose of following the asteroids and of obtaining pictures of the Milky Way. It must be inferred from this that the number of telescopic comets is much smaller than it is usually assumed to be.

Elements and Ephemeris of Giacobini's Comet (1900 *a*).—The following system of elements is based on the observation made at Nice by Javelle on February 3 and my own observations of February 16 and 21.

T 1900 April 29.0781 Greenwich Mean Time.

ω	24°	36′	56″.6	1900.0
υ	40	24	38.8	
i	146	25	22.2	

$\log q$ 0.123476

Residuals, $O - C$.

$\Delta\lambda' \cos\beta'$	− 0″.4
$\Delta\beta'$	+ 0.1

Constants for the Equator of 1900.

$x = r[9.970123] \sin(v + 79° 15′ 59″.5)$
$y = r[9.999348] \sin(v + 168 3 35.9)$
$z = r[9.559528] \sin(v + 69 57 41.8)$

Ephemeris for Greenwich Mean Midnight.

1900.	True α. h m s	True δ. ° ′	$\log \Delta$.	Br.
Feb. 26.5	2 9 52	+ 1 43.0	0.297	0.85
March 2.5	5 38	2 56.6		
6.5	2 1 53	4 7.2	0.320	0.83
10.5	1 58 34	5 15.2		
14.5	55 37	6 21.2	0.339	0.81
18.5	52 57	7 25.2		
22.5	50 32	8 27.6	0.354	0.82
26.5	48 19	9 28.8		
30.5	46 15	10 29.2	0.364	0.83
April 3.5	44 18	11 28.9		
7.5	42 26	12 28.2	0.369	0.85
11.5	40 36	13 27.5		
15.5	38 47	14 27.0	0.370	0.87
19.5	36 56	15 27.0		
23.5	35 3	16 27.8	0.366	0.91
27.5	33 6	17 29.8		
May 1.5	31 0	18 33.2	0.356	0.95
5.5	28 46	19 38.4		
9.5	26 18	20 45.7	0.342	1.00
13.5	23 36	21 55.7		
17.5	20 34	23 8.6	0.323	1.06
21.5	17 9	24 25.0		
25.5	13 16	25 45.6	0.299	1.14
29.5	1 8 47	27 10.6		
July 1.5	23 40 41	42 11.3	0.131	1.81
Aug. 1.5	19 33 28	+ 41 14.4	0.077	1.68

The unit of brightness is that on February 3.

The comet is between 10th and 11th magnitude and has a faint nucleus.

Mount Hamilton, California. C. D. PERRINE.
1900 March 2.

New Asteroid 1900 FA.—This was discovered by Charlois at Nice, Feb. 22, $9^h\ 30^m.0$, Nice, M. T., R. A, $9^h\ 30^m\ 44^s$; Dec. $+22°\ 32'$. Its daily motion was west 17′ and north 1′, and its brightness 12.2^m.

EPHEMERIS OF ASTEROID 1899, EY.

[Continued from March number.]

		R. A.	Decl.	Log r.	Log Δ.	Mag.
April	2	$5^h\ 03^m\ 09^s$	$+24°\ 10.1$	0.4565	0.4982	11.0
	4	5 49	24 18.9			
	6	8 32	24 27.5	0.4568	0.5052	
	8	11 17	24 35.9			
	10	14 05	24 44.1	0.4572	0.5119	
	12	16 55	24 52.1			
	14	19 47	24 59.8	0.4575	0.5183	
	16	22 41	25 07.4			
	18	25 37	25 14.7	0.4578	0.5245	
	20	28 35	25 21.7			
	22	31 35	25 28.5	0.4582	0.5304	
	24	34 37	25 35.0			
	26	37 41	25 41.3	0.4585	0.5361	
	28	40 47	25 47.3			
	30	43 54	25 53.1	0.4589	0.5415	
May	2	47 03	25 58.6			
	4	50 13	26 03.8	0.4592	0.5466	
	6	53 24	26 08.8			
	8	5 56 37	$+26$ 13.5	0.4596	0.5514	11.3

Elements and Ephemeris of Asteroid (434) Hungaria.—In *Astronomische Nachrichten* No. 3624, Dr. A. Berberich gives elements of this planetoid depending upon 29 observations in 1898. The elements represent the observations very closely.

ELEMENTS.

Epoch March 5.0, 1900, Berlin Mean Time.

$M = 244°\ 34'\ 15''.3$
$\omega = 122\ 55\ 42.3$
$\Omega = 174\ 39\ 17.4$
$i = 22\ 30\ 32.0$
} 1900.0

$\varphi = 4°\ 15'\ 30''.9$
$\mu = 1308''.6777$
$\log a = 0.2887826$

EPHEMERIS.

Berlin Midnight.		R. A.	Decl.	Log r.	Log Δ
		h m s	° ′		
April	4	11 51 34	$+9$ 26.3	0.2988	0.0110
	6	50 05	10 05.1		
	8	48 41	10 42.5	0.2981	0.0172
	10	47 23	11 18.3		
	12	46 11	11 52.4	0.2973	0.0248
	14	45 06	12 24.7		
	16	44 07	12 55.3	0.2966	0.0337
	18	43 15	13 24.0		
	20	42 32	13 50.8	0.2958	0.0436
	22	41 56	14 15.7		
	24	41 28	14 38.8	0.2950	0.0542
	26	41 08	15 00.0		
	28	40 55	15 19.5	0.2943	0.0654
	30	40 50	15 37.2		
May	2	11 40 54	15 53.1	0.2935	0.0771

The asteroid will be of about the twelfth magnitude during this month.

GENERAL NOTES.

We are again about on time in mailing this month's number, and we expect hereafter to publish early enough, so that all home subscribers will receive POPULAR ASTRONOMY before the first day of the month for which it is published. Foreign subscribers should be reached during the first week of the same month.

The Origin of the Lunar Surface Formations.—We have given considerable space this month and last to Professor William H. Pickering's studies of the origin of the lunar surface formations recently published in the Annals of Harvard College Observatory. His results are very suggestive and instructive.

Variation of Latitude at New York.—Professor J. K. Ree's valuable paper on the variation of latitude at New York is so well and plainly written, that those who have no experience in this kind of delicate astronomical work will readily see from it how the wonderful facts it discloses have been learned.

Professor C Piazzi Smyth.—The well known Professor C. Piazzi Smyth, for forty years Astronomer Royal of Scotland and professor of astronomy in Edinburgh, died on February 21, at the age of 81 years. When our publication of the *Sidereal Messenger* began in 1882, Professor Smyth was the first from Europe to extend congratulations. His genial letter is well remembered to this day, after nearly two decades of years.

Photographic Search for Intra-Mercurial Planets.—Being personally interested in the settlement of the intra-Mercurial planet question, the announcement that Professor W. H. Pickering is going to visit the path of totality of the eclipse of May 28, with a peculiar and ingenious photographic device for the purpose of searching for them photographically, pleases me greatly. My faith that the two objects seen by me at the Denver eclipse in 1878 were intra-Mercurial planets, has never been weakened by the failure of others to find them at subsequent eclipses, for, having no faith in their existence, no one with a proper eye-piece has made the attempt.

At the time, I hastily estimated that their direction from the sun was s. w. Since then I have visited the precise spot where my telescope was planted, to get, with perhaps greater accuracy, the directions of the four cardinal points. My conclusion was that s. of w. was a better estimate than s. w. I hope Bailey's beads will be observed with care, for their cause is a mystery. At Denver they extended half around the moon, at each end appearing like minute angular dots, increasing in size to the middle, but all of the same shape, and had one-half of them been bent over and superimposed on the other, they would have agreed in number, size and shape. At another total eclipse, in Northern California, they bore not the least resemblance to any previously seen, appearing like the letter a, of the Morse telegraphic alphabet, a dot and a dash, thus, .— .— .— extending half around the moon's circumference. I admit that at the Denver eclipse they appeared like notches in the Moon's limb, but she has no such sinuosities of elevations and depressions as to present the appearance observed.

LOWE OBSERVATORY, Echo Mountain, Cal. LEWIS SWIFT.

The Variable Star *o* Ceti (Mira) 1899.—During the past maximum of *o* Ceti only about thirty observations were secured, owing to a greater frequency of cloudy nights, and interruption by sickness and death in the family.

The observations indicate that the maximum occurred between September 18th and 25th, 1899, when the variable was only about the 4th magnitude.

Compared with the brilliant maximum of 1898, the contrast was quite noticeable in the recent apparition of this "wonderful" variable.

Date. 1899.	Central Time.	Mag.	Remarks.
August 28th	2:15 A. M.	4.3	First observation of Mira for the present apparition. Moon in last quarter. Variable is not as bright as Delta Ceti.
September 1st	2:00 A. M.	4 3	No decided change.
2d	11:15 P. M.	4.3	No change.
4th	12:10 A. M.	4.3	Cannot detect much change yet.
6th	1:00 "	4.2	A trifle brighter.
10th	2:30 "	4.1	About same as δ Ceti now.
11th	2:30 "	4.1	About same as δ Ceti, possibly trace brighter.
12th	3:00 "	4.1	No apparent change.
14th	2:00 "	4.1	No change.
18th	3:45 "	4.0	Nearly full bright Moon. Mira appears about same as α Piscium, certainly brighter than δ now.
21st	3:00 "	——	Brilliant moonlight, too near for careful comparison, but Mira appears much brighter than on 18th but this is probably owing to a tendency to estimate this variable much brighter in strong moonlight.
25th	12:05 "	4.0	About same as α Piscium.
25th	10:15 P. M.	4.0	Same as morning observation.
27th	1:00 A. M.	4.2	Trace fainter than δ Ceti this morning.
28th	10:00 P. M.	4.2	Slightly fainter than δ.
29th	10:00 "	4.3	Not as bright as δ Ceti, slightly brighter than ξ^2 Ceti.
October 2d	3:00 A. M.	4 4	Same as ξ^2 Ceti now.
4th	11:00 "	4.4	No change.
5th	10:30 "	4.3	Seems brighter than ξ^2 Ceti tonight.
11th	10:00 "	4 6	Brighter than Nu Ceti, not as bright as ξ^2 Ceti.
13th	10:00 "	4.6	Moon in first quarter. No change.
16th	11:00 "	4.5	Moon nearly full, and variable appears brighter, would estimate it as but a trace fainter than ξ^2 Ceti tonight.
29th	9:30 "	5.4	Good observation. Not as bright as V Ceti.
30th	10:45 "	5 4	Very good observation, same as last night.
November 1st	10:00 "	5.4	No apparent change.
2d	10:00 "	5.4	Slightly inferior to V Ceti.
4th	10:30 "	5.4	No change.
5th	10:00 "	5.4	Good observation, no change.
19th	7:30 "	6.5	About same as 71 Ceti now.
1900			
January 21st	6:30 "	8.5	Good observation, Mira is now about same magnitude as star of 8.5 mag. in same field of view with 4 inch telescope and $\frac{3}{4}$" eye-piece, according to Durchmusterung chart.

Comparison stars used,—δ Ceti = 4.13 ξ^2 = 4.4 α Piscium = 4.0
71 Ceti = 6.5 Nu Ceti = 4.9 V Ceti = 5.06

ALTA, IOWA, March 15th, 1900. DAVID E. HADDEN.

Report of the Board of Visitors.—We have received an anonymous communication with the above title, which is a malignant attack on the report of the Board of Visitors, who were appointed to consider the present organization and management of the United States Naval Observatory, a few months ago. The report was prepared after much deliberation by the Board of Visitors—Messrs. Comstock, E. C. Pickering and Hale, and seemed to be a fair and reasonable document, meeting the wants of the situation as well as could be expected under the circumstances. It was not, in all respects, what we had hoped it would be, yet, probably the plan proposed is a better one for the present time.

We do not believe there is any real ground for the shameful attack which has been made upon the report, and the visitors personally, by some one who, presumably, knows the situation well, and has taken this way to neutralize the manifest influence of this movement, which will certainly succeed in due time.

The author of this attack has industriously circulated it at home and abroad, and this is the reason we give it any attention. We know all the visitors personally, and we believe them to be gentlemen wholly above insinuations of this kind. As for the anonymous writer of those charges, it is best for him that he keep in the dark.

The Errors of Star Photographs.—Three significant papers by H. H. Turner, Savilian Professor at Oxford, England, published some time ago in the Monthly Notices of the Royal Astronomical Society, have just come to hand as reprints of those articles. The full title of the first and second papers is "On the Errors of Star Photographs Due to Optical Distortion of the Object Glass with Which the Photograph is Taken." Of the third, "On the Curvature of Star Trails on the Photographic Plates as a Means of Investigating Optical Distortion."

It will be remembered that in 1887 an International Conference assembled at Paris to consider the construction of a chart of the heavens by photography. Almost the first question asked was, "With what kind of an instrument should the work be done?" The reflector, the refractor with acromatic object glass and the photographic doublet were fully considered, and the simple photographic refractor was finally chosen. No astronomer in 1887 knew much about astronomical photography, and the best in the conference were timid in the choice of the kind of telescope to be recommended, or the methods of work by which this large task should be prosecuted. The fears about optical distortion were general, and early opportunities have been sought to test the real value of photographic work in this and other essential particulars. These papers bring out quite satisfactory results in an exhaustive examination of the Oxford plates and those by Professor E. C. Pickering, of Harvard College Observatory, taken with Bruce photographic doublet at Arequipa.

The gratifying results of these papers are, that the early fears about distortion, which caused much discussion in the outset, are found to be largely groundless, and the advantages of getting a large field are now claimed to be "too obvious to need explanation." This is almost unexpected evidence of the high precision of the modern photographic telescope, as well as increased confidence in photographic methods in stellar astronomy.

Total Eclipse of the Sun May 28, 1900.—A useful pamphlet of 32 pages published under the direction of the Superintendent of the U. S. Naval Observatory, Capt. C. H. Davis, has just been received. It contains suggestions

and data to assist in the observation of the total solar eclipse of May 28, 1900. It was prepared by Professor H. D. Todd, Director of the Nautical Almanac, and S. J. Brown Astronomical Director of the Naval Observatory.

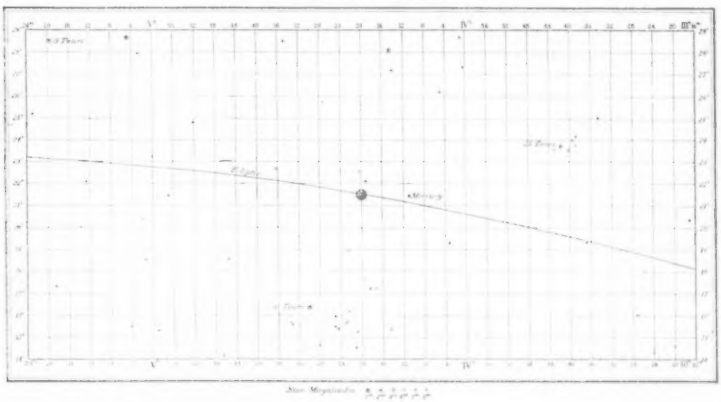
The pamphlet treats quite fully of

1. Sketches of the corona with the naked eye.
2. The Moon's shadow bands.
3. Photographs of the corona, and
4. Telescopic oservations.

The last part contains the ordinary astronomical data with a series of very good charts as follows:

1. A plate of the geographical limits and path of totality as taken from the Nautical Almanac.

2. A large chart of the stars and planets in the neighborhood of the Sun during the time of totality as given in reduced scale below:



3. A large chart showing the path of totality across Mexico and Texas with the hours of progress printed in red.

4. A large chart showing the path of totality across the United States, through Louisiana, Mississippi, Alabama, Georgia, the Carolinas and Virginia.

It is reported that extensive preparations are being made at the United States Naval Observatory for photographing and observing the total eclipse of the Sun. Astronomical Director D. J. Brown said recently: "If the weather is favorable we expect to secure some fine observations of the phenomena."

Two government stations will observe the eclipse, one in South Carolina and one in Georgia. There may be a branch station as far south as Union Springs, in Alabama, a place directly under the line of totality.

Professor Brown said that the Observatory is asked many questions in regard to the eclipse, indicating the wide-spread interest manifested in scientific circles in the approaching event.

A party of Eastern scientists will come to Washington some days prior to the 28th of May and will accompany the Observatory corps South, some going to Georgia and others remaining in South Carolina.

Brief Review of Solar Observations September to December 1899.—(Continuation from October 1899. P. A.) SEPTEMBER. The marked decrease in the number and size of Sun spots, and the frequency of days without spots noted in August, continued during the first three weeks of September, with the exception of occasional sporadic pores on five or six days. From the 24th to the close of the month, several small but active groups were on the disc, one of which suddenly appeared, and another rapidly increased between the 25th and 26th, this outbreak was coincident with a brief display of Aurora on the evening of the 25th.

OCTOBER.—There were eighteen spotless days during the month of October. No spots were visible during the first three weeks, except a few dots, but on the 23rd a fairly large and interesting group appeared at the East limb. With the spectroscope the region between the large spots of the group was much disturbed, the Ha line being reversed at many points. Unfortunately cloudy weather prevented further observations until the 29th, when the group was a small train and fast diminishing, being reduced to a mere dot as it approached the West limb.

NOVEMBER.—Sun spots were small, few and uninteresting during the month of November. There were only eight spotless days, all of which were in the first decade.

DECEMBER.—Owing to sickness and death in the family, and more or less cloudiness, observations were seriously interrupted this month. Between the 12th and 20th a couple of small groups were present, but they faded out before reaching the West limb, and the disc was nearly free from spots during the remainder of the month.

The following table exhibits the numerical results of the observations for the above four months:

Months. 1899.	Number of Observing Days.	Average Number of Groups.	Spots.	Faculæ.	Average Number Groups North Latitude.	Groups South Latitude.
September	27	0.8	3.4	1.4	0.1	0.7
October	22	0.5	4.3	1.4	0.1	0.4
November	18	0.9	3.3	0.9	0.3	0.6
December	9	1.0	2.8	1.2	0.4	0.6

Compared with the year 1898, there has been a decided diminution in the average number of groups, spots and faculae during the past year, while the total number of spotless days has increased from 30 in 1898 to 108 in the year 1899.

In the table below is given the annual average results for the years 1891 to 1899, inclusive:

Years.	Number of Days of Observation.	Average Annual Number of Groups.	Spots.	Faculæ.	Total Number of Spotless Days.
1891	257	2.9	14.9	3.6	24
1892	205	5.6	34.0	4.1	0
1893	177	6.6	36.6	4.1	0
1894	139	5.6	30.0	3.4	0
1895	149	5.2	30.5	3.5	0
1896	197	3.2	17.8	2.9	5
1897	198	2.2	11.0	2.3	29
1898	234	2.1	11.0	2.4	30
1899	259	1.1	4.8	1.5	108

ALTA, IOWA, March 20th, 1900. DAVID E. HADDEN.

Queries and Short Answers.—5. If a man celebrates his twentieth birthday when he is nineteen years old to the minute, why should he not celebrate the twentieth century the moment we are 1900 years from the given starting point? H. M. S.

Answer. He certainly should celebrate the beginning of the twentieth century when "we are 1900 years from the given starting point." The trouble seems to be to get the starting point right. The first year of the first century did not end until that year was completed; the last year of that century was not completed until the end of the 100th year. So, also, the end of the 1900th year will be the end of the 19th century. If our querist will read the article on page 140, March number, he will see how to begin the reckoning.

6. What name is given by astronomers to the spectrum of the layer of gases immediately above the photosphere of the Sun? S. G.

Answer. It is sometimes called the "flash" spectrum, because of the sudden change of the ordinary solar spectrum to one of bright lines. The discovery of this important fact was made by Professor C. A. Young of the Princeton University in 1870 while observing the Spanish eclipse of the Sun. He says, (article 319 of his General Astronomy) "The lines of the solar spectrum, which up to the final obscuration of the Sun had remained dark as usual, with the exception of a few belonging to the spectrum of the chromosphere, were suddenly "reversed," and the whole length of the spectrum was filled with brilliant-colored lines which flashed out quickly and then gradually faded away in about two seconds —a most beautiful thing to see. The name, *flash*, probably came from the suddenness of the reversal of the lines, and the use of the word by Professor Young n the description of this singularly beautiful sight.

7. Who is the publisher of Wm. Peck's Constellations?

Answer. About information of the book, Wm. Peck's Constellations, I can say that I bought, a few years ago, of G. P. Putnam's Sons, 27 and 29 West 23 St., N. Y., a copy of the Popular Handbook and Atlas of Astronomy by William Peck, Astronomer, Edinburgh. In the preface to this book Mr. Peck says: "In a condensed form there is presented the author's investigation as to the origin of the constellations, which at some future time he hopes to elaborate in a special work. C. A. H.

The inquiry is probably for William Peck's Constellations, a book which Silver, Burdett & Co., of Boston handle. This is a thin book.

In Mayer and Miller's Catalogue No. 169, No. 1359 is Peck, W. The observers Atlas of the Heavens containing catalogues of the accurate positions, magnitudes, etc., of over 1400 double stars, star clusters or nebulae, etc., London, 1898. Mks 21. This is evidently Peck's larger work. H. A. H.

We do not publish any book called "Constellations" by William Peck. We have, by him, A Popular Handbook and Atlas of Astronomy, designed as a complete guide to a knowledge of the heavenly bodies, and as an aid to those possessing telescopes, containing 44 large plates, and numerous illustrations, diagrams, etc., price $5.50, net. This we can supply, if desired.

G. P. PUTNAM'S SONS.

Memoria Sobre el Eclipse Total de Sol.—The Astronomical Observatory of Madrid, Spain, has issued a pamphlet of 107 pages, for benefit of those who are to observe the total eclipse of May 28. It is provided with beautifully colored charts and suggestions, directions and useful data for the contemplated observations. The pamphlet is a creditable piece of work.

On a Class of Particular Solutions of the Problem of Four Bodies, is the title of an interesting mathematical paper recently published by F. R. Moulton, of the University of Chicago. It is encouraging to see the young men pressing the mathematics into service in the needed astronomical investigations when astrophysics can do very little.

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